Math 244 – Sample Exam 1 – Professor Feehan
(100 points – 1 hour and 20 minutes)

1 (20 points)**.** Consider the initial value problem $ty' + 2y = 4t^2$, $y(1) = 2$.
(a) What type of first-order differential equation is this?
(b) What is the interval on which the solution exists?
(c) Find the solution.

2 (10 points)**.** Consider the equation $y' + y^2 \sin x = 0$.
(a) What type of first-order differential equation is this?
(b) Find a solution other than $y = 0$.

3 (15 points)**.** Consider the equation $t^2y' + 2ty - y^3 = 0, t > 0$.
(a) What type of first-order differential equation is this?
(b) Find a solution other than $y = 0$. [Hint: Use the substitution $v = y^{-2}$ and first solve the resulting equation for v.]

4 (20 points)**.** Consider the equation $y' = \dfrac{y}{y \sin y - x}$.
(a) When is an equation of the form $M(x, y) + N(x, y)y' = 0$ exact?
(b) Is the given equation exact? Explain using your answer to (a).
(c) Solve the given equation. [Hint: If it is not exact, use an integrating factor of the form $\mu(y)$ to solve a related exact equation.]

5 (15 points)**.** Euler's method can be used to approximate the solution to first-order differential equations of the form $y' = f(t, y)$ at times t_n by a sequence of solutions y_n to the first-order difference equation

$$y_{n+1} = y_n + f(t_n, y_n)h, \quad y_0 = y(0), \quad n = 0, 1, 2, \ldots,$$

where $h = t_{n+1} - t_n$ is the step size.

Use Euler's method to find approximate values to the initial value problem

$$y' = 5(t + 1) - 3\sqrt{y}, \quad y(0) = 1$$

at times $t = 1, 2$ using $h = 1$. It is not necessary to simplify your answer for y_2.

6 (20 points)**.** Consider the initial value problem $y'' + y' - 2y = 0$, $y(0) = 1$, $y'(0) = 1$.
(a) Find two solutions, y_1, y_2, to the equation $y'' + y' - 2y = 0$.
(b) Compute the Wronskian determinant $W(y_1, y_2)$ of the solutions you found in (a). What property must $W(y_1, y_2)$ satisfy in order that y_1, y_2 form a fundamental set of solutions?
(c) Find the solution to the given initial value problem.