Comments on Section 7.2

The textbook uses formulas from the table on page 410 to evaluate the integrals of examples 3, 4, 6, 7, 8 of section 7.2. The student is expected to use these same formulas to do the exercises 8, 11, 20, 21, 30, 42 from the list of suggested homework exercises in the syllabus for 7.2. This is reasonable when you have an open book in front of you. However, the Math 152 exams are closed book. Formula sheets are not allowed during Math 152 exams. In other words, the formulas listed in the table on page 410 are simply not available to you during exams.

If problems like examples 3, 4, 6, 7, 8 and exercises 8, 11, 20, 21, 30, 42 of 7.2 appear on an exam, there is a way to do the problems without consulting a table of integration formulas. For example, the integral of $\sec^3 x$ can be done using an integration by parts and a trigonometric identity. First, we write

$$\int \sec^3 x \, dx = \int \sec x \sec^2 x \, dx$$

$$= \sec x \tan x - \int (\sec x \tan x) \tan x \, dx$$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$= \sec x \tan x - \int \sec^3 x - \sec x \, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx.$$

Adding $\int \sec^3 x \, dx$ to both sides, we obtain

$$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x \, dx.$$

Division by 2 leads to the answer

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C.$$

The textbook shows you in 7.2 how to integrate $\sec x$. By the way, the integration of $\csc x$ is very similar.

The method shown above (integration by parts and then a trigonometric identity) can be used to derive formulas 11, 12, 13, 14, 20, 22 in the table on page 410. For example: We
do the computation

\[ \int \sin^n x \, dx \]
\[ = \int \sin^{n-1} x \sin x \, dx \]
\[ = (\sin^{n-1} x)(-\cos x) - \int (n-1)(\sin^{n-2} x)(\cos x)(-\cos x) \, dx \]
\[ = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \cos^2 x \, dx \]
\[ = -\sin^{n-1} x \cos x + (n-1) \int (\sin^{n-2} x)(1-\sin^2 x) \, dx \]
\[ = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx, \]

add \((n-1)\int \sin^n x \, dx\) on both sides to obtain

\[ n \int \sin^n x \, dx = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \, dx, \]

and finally divide by \(n\) to get

\[ \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx. \]

Integrals like \(\int \sin 4x \cos 3x \, dx\) in Example 8 of 7.2 can be done in a way that is conceptually equivalent to the method in Example 5 of section 7.1. It involves two integrations by parts. The only additional difficulty is the introduction of some extra arithmetic. We would write

\[ \int \sin 4x \cos 3x \, dx \]
\[ = (\sin 4x) \left( \frac{\sin 3x}{3} \right) - \int (4 \cos 4x) \left( \frac{\sin 3x}{3} \right) \, dx \]
\[ = \frac{1}{3} \sin 4x \sin 3x - \frac{4}{3} \int \cos 4x \sin 3x \, dx. \]

Then we would rewrite the last integral:

\[ \int \cos 4x \sin 3x \, dx \]
\[ = (\cos 4x) \left( -\frac{\cos 3x}{3} \right) - \int (-4 \sin 4x) \left( -\frac{\cos 3x}{3} \right) \, dx \]
\[ = -\frac{1}{3} \cos 4x \cos 3x - \frac{4}{3} \int \sin 4x \cos 3x \, dx. \]
Combining these two displayed computations, we get

\[ \int \sin 4x \cos 3x \, dx = \frac{1}{3} \sin 4x \sin 3x + \frac{4}{9} \cos 4x \cos 3x + \frac{16}{9} \int \sin 4x \cos 3x \, dx. \]

Now we subtract \( \frac{16}{9} \int \sin 4x \cos 3x \, dx \) from both sides. This gives

\[ -\frac{7}{9} \int \sin 4x \cos 3x \, dx = \frac{1}{3} \sin 4x \sin 3x + \frac{4}{9} \cos 4x \cos 3x + C, \]

which can be multiplied by \(-9/7\) to get the answer

\[ \int \sin 4x \cos 3x \, dx = -\frac{3}{7} \sin 4x \sin 3x - \frac{4}{7} \cos 4x \cos 3x + C. \]

One can use trigonometric identities to show that this answer is the same as the answer that would be obtained with the use of formula 24 in the table on page 410.