Formula sheet for the final exam in Math 135

Limits. $\lim_{x\to a} f(x) = L$ is equivalent to $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$.

Continuity. f is continuous at a when all of the following conditions are satisfied:
1. $f(a)$ is defined, 2. $\lim_{x\to a} f(x)$ exists, 3. $\lim_{x\to a} f(x) = f(a)$.

Differentiability. f is differentiable at x when $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ exists. The number $f'(a)$ is the slope of the tangent line to the graph of f at a.

Differentials. $dy = f'(x)dx$, $\Delta y = f(x+\Delta x) - f(x)$, $f(a+\Delta x)$ is approximately equal to $f(a) + f'(a)dx$ when $dx = \Delta x$.

Graphing and optimization. **Relative max and min** of a function f can occur only at **critical points** (points x in the domain of f where $f'(x) = 0$ or $f'(x)$ does not exist). **Absolute max and min** occur only at critical points or endpoints. **Inflection points** are points where the concavity of f changes sign. The line $x = a$ is a **vertical asymptote** of the graph of a function f if $\lim_{x\to a^+} f(x) = +\infty$ or $-\infty$, or if $\lim_{x\to a^-} f(x) = +\infty$ or $-\infty$. The line $y = b$ is a **horizontal asymptote** of the graph of f if $\lim_{x\to +\infty} f(x) = b$ or $\lim_{x\to -\infty} f(x) = b$.

Basic log and exp laws. $e^{\ln x} = x$ for $x > 0$ $\quad \ln(e^x) = x$ for all x

$$\ln(ab) = \ln a + \ln b \qquad \ln(a/b) = \ln a - \ln b \qquad \ln(a^b) = b\ln a \qquad \ln 1 = 0 \qquad \ln e = 1$$

$$e^a e^b = e^{a+b} \qquad e^a/e^b = e^{a-b} \qquad (e^a)^b = e^{ab} \qquad e^0 = 1 \qquad e^1 = e$$

Basic trig identities. $\sin^2 x + \cos^2 x = 1 \quad \sin(-x) = -\sin x \quad \cos(-x) = \cos x$

$$\sin(x+2\pi) = \sin x \qquad \cos(x+2\pi) = \cos x \qquad 360 \text{ degrees} = 2\pi \text{ radians}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

The indefinite integral. If $F'(x) = f(x)$ then F is an antiderivative of f. $\int f(x)\,dx$ is the set of all antiderivatives of f. $F'(x) = f(x)$ is equivalent to $\int f(x)\,dx = F(x) + C$, where C is a constant.

The definite integral. If f is continuous and $f(x) \geq 0$ on $[a,b]$ then $\int_a^b f(x)\,dx$ is the area of the region between $y = f(x)$, the x-axis, $x = a$ and $x = b$. If $f(x) < 0$ occurs for some x then $\int_a^b f(x)\,dx$ is the area above $[a,b]$ minus the area below $[a,b]$. The fundamental theorem of calculus says $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f.

Riemann sums. Suppose f is defined on the interval $[a, b]$. Consider a subdivision of $[a, b]$ into n subintervals of length $\Delta x = (b - a)/n$. Let $x_1, x_2, \ldots, x_n$ be representative points in these subintervals. The number $f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$ is called a Riemann sum.

Average value. The average value of f over $[a, b]$ is $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.

Differentiation rules	**Integration rules**
$\frac{d}{dx}(kx) = k$ for constant k	$\int k\,dx = kx + C$ for constant k
$\frac{d}{dx}[kf(x)] = kf'(x)$ for constant k	$\int kf(x)\,dx = k\int f(x)\,dx$ for constant k
$\frac{d}{dx}(x^k) = kx^{k-1}$ for constant k	$\int x^k\,dx = \frac{x^{k+1}}{k+1} + C$ for constant $k \neq -1$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$

Integration by substitution. If $u = g(x)$ then $\displaystyle\int f(g(x))g'(x)\,dx = \int f(u)\,du$ and $\displaystyle\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Chain Rule. If $h(x) = g(f(x))$ then $h'(x) = g'(f(x))f'(x)$ when f, g are differentiable.

Product and Quotient Rules. $(fg)' = f'g + fg'$ and $(f/g)' = (f'g - fg')/g^2$.

Calculus in economics. Profit P equals revenue R minus total cost C. If the unit price is p and the number of units is x then $R = px$. The average cost is $C(x)/x$. The demand equation relates p and x. If $x = f(p)$ then $E(p) = -\dfrac{pf'(p)}{f(p)}$ is the elasticity of demand. The demand is elastic if $E(p) > 1$. The demand is inelastic if $E(p) < 1$. The demand is unitary if $E(p) = 1$.

Exponential functions as mathematical models. Problems about exponential growth and decay use the formulas $Q(t) = Q_0 e^{kt}$ and $Q(t) = Q_0 e^{-kt}$, respectively.