Formula sheet for Exam I, Math 135

One-sided limits. If $f(x)$ approaches L as x approaches a from the right (keeping $x \neq a$), then we say $\lim_{x \to a^+} f(x) = L$. If $f(x)$ approaches L as x approaches a from the left (keeping $x \neq a$), then we say $\lim_{x \to a^-} f(x) = L$.

Relation between limits and one-sided limits. If we have $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$ (same L) then we conclude $\lim_{x \to a} f(x) = L$. If $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$ then $\lim_{x \to a} f(x)$ does not exist.

Continuity. The function f is continuous at a when all three of the following conditions are fulfilled: $f(a)$ is defined, $\lim_{x \to a} f(x)$ exists, $\lim_{x \to a} f(x) = f(a)$.

Existence of zeros. If f is continuous on $[a, b]$ and $f(a)$, $f(b)$ have opposite signs, then the equation $f(x) = 0$ has at least one solution in the interval (a, b).

Intermediate Value Theorem. If f is continuous on $[a, b]$ and M lies between $f(a)$ and $f(b)$, then the equation $f(x) = M$ has at least one solution in the interval $[a, b]$.

The derivative. If the limit $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ exists then this limit is called $f'(x)$. If $f'(a)$ exists then $f'(a)$ is the slope of the tangent to the curve $y = f(x)$ at the point $(a, f(a))$. The *product and quotient rules* say $(fg)' = fg' + gf'$, $(f/g)' = (gf' - fg')/g^2$, respectively, for differentiable functions f and g. If r is a constant then the derivative of x^r is rx^{r-1}. In addition, differentiable functions have the properties $(f + g)' = f' + g'$ and $(cf)' = cf'$ when c is a constant.

The chain rule. If f, g are differentiable functions and $h(x) = g(f(x))$ then the chain rule says

$$h'(x) = g'(f(x))f'(x) \; .$$

If we write $u = f(x)$ and $y = g(u)$ (which implies $y = g(f(x)) = h(x)$) then the chain rule can be rewritten in the form

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \; .$$

Velocity. If $f(t)$ is the position of an object at time t then $f'(t)$ is its velocity at time t. If $t_1 < t_2$ are two different values of t then $\dfrac{f(t_2) - f(t_1)}{t_2 - t_1}$ is the average velocity over the interval $[t_1, t_2]$.

Calculus in economics. Here p is the price per unit and x is the number of units. If $x = f(p)$ is the demand function then the elasticity of demand is $E(p) = -\dfrac{pf'(p)}{f(p)}$.

$$\text{The demand is} \begin{cases} \text{elastic} & \text{if } E(p) > 1, \\ \text{unitary} & \text{if } E(p) = 1, \\ \text{inelastic} & \text{if } E(p) < 1. \end{cases}$$

If $C(x)$ is the total cost of producing x units then the average cost function is $\overline{C}(x) = C(x)/x$.

Geometry. For a sphere of radius r, the volume is $(4/3)\pi r^3$ and the surface area is $4\pi r^2$. For a cylinder, the volume is $\pi r^2 h$, where r is the radius of the base and h is the height.