Formula sheet for Exam II, Math 135 Fall 2002

If $y = f(x)$, its **differential** is $dy = f'(x)dx$. This differential is approximately equal to $\Delta y = f(x + \Delta x) - f(x)$ when $dx = \Delta x$. The number $f(a + \Delta x)$ is approximated by $f(a) + f'(a)\Delta x$.

The line $x = a$ is a **vertical asymptote** of $y = f(x)$ if $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$. For example: If $P(x)$, $Q(x)$ are polynomials, $f(x) = P(x)/Q(x)$, $Q(a) = 0$ but $P(a) \neq 0$ then $x = a$ is a vertical asymptote of $y = f(x)$.

The line $y = b$ is a **horizontal asymptote** of $y = f(x)$ if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

A function f is **increasing** on intervals where f' is positive. It is **decreasing** on intervals where f' is negative. It is **concave up** on intervals where f'' is positive. It is **concave down** on intervals where f'' is negative.

A **critical point** of a function is any point x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist.

To find the absolute maxima and the absolute minima of a continuous function on the interval $[a, b]$: Evaluate the function at a, b and all of its critical points between a and b. Determine the max and min of these evaluations.

First derivative test at a critical point c of f: f has a relative maximum at c if the sign of $f'(x)$ goes from $+$ to $-$ as x moves across c from left to right. f has a relative minimum at c if the sign of $f'(x)$ goes from $-$ to $+$ as x moves across c from left to right.

Second derivative test at a critical point c of f: If $f'(c) = 0$ and $f''(c) < 0$ then f has a relative maximum at c. If $f'(c) = 0$ and $f''(c) > 0$ then f has a relative minimum at c.

The function f has an inflection point at c if the concavity of $y = f(x)$ changes as x moves across c.

Basic log and exp laws: $e^{\ln x} = x$ for $x > 0$, $\ln(e^x) = x$ for all x,

$$\ln(ab) = \ln a + \ln b\ , \quad \ln(a/b) = \ln a - \ln b\ , \quad \ln(a^b) = b\ln a\ , \quad \ln 1 = 0\ , \quad \ln e = 1,$$

$$e^{a+b} = e^a e^b\ , \quad e^{a-b} = e^a/e^b\ , \quad (e^a)^b = e^{(ab)}\ , \quad e^0 = 1\ , \quad e^1 = e$$

Basic trig identities: $\sin^2 x + \cos^2 x = 1\ , \quad \sin(-x) = -\sin x\ , \quad \cos(-x) = \cos(x),$

$$\sin(x + 2\pi) = \sin x\ , \quad \cos(x + 2\pi) = \cos x,$$

$$\tan x = \frac{\sin x}{\cos x}\ , \quad \cot x = \frac{\cos x}{\sin x}\ , \quad \sec x = \frac{1}{\cos x}\ , \quad \csc x = \frac{1}{\sin x}$$

Basic differentiation rules for trig, log and exp functions:

$$\frac{d}{dx}[\sin(f(x))] = [\cos(f(x))]f'(x)\ , \quad \frac{d}{dx}[\cos(f(x))] = -[\sin(f(x))]f'(x),$$

$$\frac{d}{dx}[\tan(f(x))] = [\sec^2(f(x))]f'(x)\ , \quad \frac{d}{dx}[\sec(f(x))] = [\sec(f(x))][\tan(f(x))]f'(x),$$

$$\frac{d}{dx}\left[e^{f(x)}\right] = \left[e^{f(x)}\right] f'(x)\ , \quad \frac{d}{dx}[\ln(f(x))] = f'(x)/f(x)\ , \quad \frac{d}{dx}[\ln|f(x)|] = f'(x)/f(x)$$