Review problems for the first midterm exam in Calculus 135 - Fall 2002

NOTE: These are only practice problems! Your exam may have types of problems that are not represented on this sheet. It is your responsibility to study **all** of the material and to master **all** of the homework problems. The answers to the questions must be exact answers. For example, the number 1.732050808 is not a correct answer when the right answer is actually $\sqrt{3}$.

1) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\infty}\sqrt{\frac{3x^2+4x+10}{5x^2+7x+8}}$ (b) $\displaystyle\lim_{x\to 2}\frac{2-x}{x^2+x-6}$ (c) $\displaystyle\lim_{x\to 2}\frac{x-2}{\sqrt{x+14}-4}$

(d) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{7x^2+x+5}}{x}$ (e) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{2x^2+4x+3}}{x}$ (f) $\displaystyle\lim_{x\to\infty}\frac{x^2+x+5}{3x^3+6x^2+3x+2}$

2) Consider the function f given below, where A and B are constants.

$$f(x)=\begin{cases} Ax+1 & \text{if } x\le 1,\\ x^2-1 & \text{if } 1<x<2,\\ x+B & \text{if } 2\le x.\end{cases}$$

What values of A and B will make f a continuous function?

3) Show that the equation $x^3+2x^2+3x+5=0$ has at least one solution on the x-axis. Do not graph this. Do not attempt to find an approximation to a solution.

4) Find the domain of $f(x)=\sqrt{x-2}+\sqrt{5-x}$.

5) Find $g(f(2))-f(g(2))$ when $f(x)=x^2+x$ and $g(x)=1/x$.

6) Assume that $2x+3y=1$ is the line tangent to the curve $y=f(x)$ at $x=4$. Find $f(4)$ and $f'(4)$.

7) Sketch the graph of some function f which has all of the following properties simultaneously:

(a) The domain of f is $(1,5)$,

(b) $\displaystyle\lim_{x\to 1^+} f(x)=0$,

(c) $\displaystyle\lim_{x\to 5^-} f(x)=2$,

(d) f is continuous on $(1,5)$,

(e) f is not differentiable at $x=3$.

8) Suppose $f(x)=1/x$. Verify the identity $f'(x)=-1/x^2$ using only the definition of the derivative as a limit.

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9) An ice sculpture has the shape of a cylinder. It is melting at the rate of 30 ft^3 per hour. Assume that it maintains its cylindrical character (with shrinking radius and height) as it melts. What is the rate of decrease of its radius when the height is 5 ft, the radius is 3 ft, and the height is decreasing at the rate of 1/2 ft per hour?

10) Find the second derivative of each of the following functions.

(a) $f(x) = \sqrt{x^2+3}$ (b) $g(x) = 1/(3x+1)$ (c) $h(x) = (x^2+x)^{10}$

11) Sketch the graph of some function f which has all of the following properties simultaneously:

(a) the domain of f is $[1, 5]$,

(b) $\lim_{x \to 2^-} f(x) = f(2) \neq \lim_{x \to 2^+} f(x)$,

(c) $\lim_{x \to 3} f(x)$ exists, but f is not continuous at $x = 3$.

(d) $f'(4) = 0$.

12) Find the derivative of each of the following functions.

(a) $f(x) = \sqrt{\dfrac{x^2+3}{x^4+3}}$ (b) $g(x) = (\sqrt{x} + 4x^3)^{10}(5x - 1/x)^{15}$

(c) $h(x) = \sqrt{4 + \sqrt{x^2+1}}$ (d) $k(x) = (x^2+3)(x^4+x+2)(x^6+x^3+1)$

13) Find the equation of the line tangent to the curve $x^2y^2 + x^3y^4 + y = 470$ at the point $(3, 2)$.

14) Assume that h and k are functions such that $h'(x) = (1+x^2)^{-1}$ and $k(x) = h(x^3)$. Find $k'(x)$.

15) Let f, g, h be functions such that $h(x) = g(f(x))$. Assume $f(2) = 5$, $f(1) = 3$, $f'(2) = -1$, $f'(1) = 8$, $g'(5) = 7$, $g'(3) = 4$. Find $h'(1)$ and $h'(2)$.

16) If p is the price in dollars for each gallon of a certain brand of paint then

$$x = 100 - p^2/9 \qquad (0 \leq p \leq 30)$$

is the number of gallons demanded per month. For what value of p is the demand unitary?