Review Problems for Exam II, Math 135, Fall 2002

NOTE: These are only practice problems! You are responsible for studying **all** the material and should be able to do **all** homework problems!

1. A bacterial population doubles every 10 days. How long does it take for this population to triple?

2. Solve each of the following equations: (a) $2^{2x} = (1/4)^{1-3x}$ (b) $e^{3x}/e^{5x} = 4$

3. The number of fish in a certain pond is given approximately by

$$Q(t) = \frac{5000}{1 + 3e^{-(t/20)}},$$

where t is the time in days.
(a) How many fish can the pond support?
(b) At what time t does the graph of Q have an inflection point?

4. What is the annual interest rate (compounded continuously) that will double funds every 10 years?

5. Use logarithmic differentiation to find $\frac{dy}{dx}$ in each of the following cases:

(a) $y = x^{\sin x}$ (b) $y = \frac{(1 + 2x^2)^{25}}{(3 + \cos x)^{11}}$

6. Find $\frac{dy}{dx}$ in each of the following cases:
(a) $y = \sec(x^2 + x^4)$ (b) $y = \cos^4(x + \tan x)$

7. Sketch a graph of a continuous and differentiable function f with the following properties: The only critical points of f are $x = 0$, $x = 2$, and $x = 6$. Further, $f''(x) = 0$ only when $x = 1$ and $x = 4$. Finally, $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = -\infty$.

8. Let $f(x) = 2x^3 - 6x^2 - 18x + 3$.

(a). Find all relative extrema of $f(x)$. Apply the second derivative test or the first derivative test to determine which are relative minima and which are relative maxima.

(b) Find the absolute maximum and absolute minimum of $f(x)$ on $[-2, 2]$.

9. An open box with a square base and a total volume of 1000 cubic inches is to be constructed from two types of materials. The base should be made of a heavy duty metal which costs \$4 per square inch. The vertical sides should be made of cardboard which costs 80 cents per square inch. Find the dimensions of the box which will minimize the total cost.
Don't forget to check that your answer **minimizes** the cost.

10. Sketch the graph of the function $f(x) = \frac{x^2 - 1}{x^2 - 4}$ using **calculus only!** It is OK to use your calculator to check your answer, but you need to say, based on calculus,

how the answers are obtained. **Show all work:** Find the domain of $f(x)$, find the x and y intercepts, find all vertical and horizontal asymptotes, find the intervals where f is increasing/decreasing, the intervals where is it concave upward/downward, the local extrema and the inflection points.

11. The **derivative** of f is $f'(x) = x^{15}(x+1)^{10}$.
(a). Find the critical points of f and check where f is increasing and decreasing.
(b). Compute $f''(x)$ and determine the x values at the points of inflection.
Note: **Do not try to find f!**

12. Find the absolute maximum and minimum of the function $f(x) = \sin x + \cos x$ on $[0, 2\pi]$.

13. Use differentials to approximate the values of $(8.01)^{1/3}$ and $(7.99)^{1/3}$.

14. The weekly quantity demanded of "Comfy" chairs is given by $p + (0.2)x^2 = 280$ where p is measured in dollars and x is the number of chairs demanded per week.
(a). Use differentials to estimate the change in the price of a unit when the weekly quantity demanded changes from 20 to 21.
(b). Determine the elasticity when $x = 20$ and $p = 200$.

15. Assume $f(x) = \tan(x^3 + 3)$. Find $f''(x)$.