Review problems for the Math 135 final exam

This review sheet covers only the sections that did not appear in the review sheets for the first and second midterm exams. The final exam is cumulative. You should study all three review sheets. Of course, you should be able to do all of the suggested homework problems.

1. Find the function f which has the properties $f'(x) = e^{2x} + \cos(3x)$ and $f(0) = 10$.

2. Find the function f such that the curve $y = f(x)$ passes through $(2,3)$ and has a slope of $x + 4$ at the point $(x, f(x))$.

3. Evaluate the following integrals.

$$\int \frac{x^3+x+1}{x^4+2x^2+4x}\,dx \qquad \int (\sqrt{x}+x)(3+\sqrt{x})\,dx \qquad \int (1+e^u)(3+e^{3u})\,du$$

$$\int \sin(3x) + \cos(4x)\,dx \qquad \int_0^{\pi/2} \sin x \cos^2 x\,dx$$

$$\int_0^{\pi} \sin^3 x\,dx \qquad \int \tan x \sec^2 x\,dx \qquad \int_1^2 \frac{1+3x^3}{x^2}\,dx$$

4. Use integration by substitution, some trigonometric identities and the fact $-\ln t = \ln(1/t)$ to show that $\int \tan x\,dx = \ln|\sec x| + C$.

5. Let R be the region bounded by $y = 16 - x^2$ and the x-axis.
a. Sketch the region R.
b. Approximate the area of R using a Riemann sum with 4 subintervals and choosing the points x_i to be the left endpoints of the subintervals.
c. Approximate the area of R using a Riemann sum with 4 subintervals and choosing the points x_i to be the right endpoints of the subintervals.
d. Find the exact value of the area of R.

6. The acceleration of a car is

$$a(t) = 4t \text{ feet/sec}^2 \qquad 0 \le t \le 20$$

where t is time measured in seconds. At time $t = 0$ the position of the car is 2 feet. At time $t = 0$ the velocity of the car is 3 feet/sec. What is the position of the car at time t?

7. Find the area between the curve $y = 1 + |x|$, the x-axis and the lines $x = -2$ and $x = 3$.

8. Consider the definite integral $\int_{-2}^{2} x\sqrt{1+x^2}\,dx$.
a. Evaluate this integral using a substitution.
b. Evaluate this integral using only a symmetry argument.
c. Evaluate $\int_{-2}^{2} x\sqrt{1+x^4}\,dx$ using an argument similar to the argument in b.