Review Problems for Exam 1 in Math 135 - Fall 2004

NOTE: Your exam may have types of problems that are not represented below. The choice of midterm exam questions is up to the individual instructor. In order to be prepared for many types of exam questions, you should study all of the textbook material and make sure you can do all of the suggested homework problems on the course web page. Answers to exam questions must be exact, not calculator approximations. For example, the number 1.732050808 is not a correct answer when the right answer is actually $\sqrt{3}$.

(1) Solve the inequality $|3x + 4| > 2$.
(2) Solve the inequality $|2x + 1| \geq |3x - 1|$. Hint: Square both sides.
(3) Find the center and the radius of the circle $2x^2 - 2x + 2y^2 + 8y - 8 = 0$.
(4) Assume the following: $L_1$ is the line that passes through $(1, 2)$ and $(3, 5)$; $L_2$ is a line that contains $(4, -1)$; $L_1$ and $L_2$ are perpendicular to each other. Find equations for $L_1$ and $L_2$.
(5) Consider the functions $f(x) = x^7 + 1$, $g(x) = \sqrt{x^8 + 3}$, $u(x) = \sin^5(3x)$, $v(x) = \sin(x^4 - 7)$. Which of these functions are odd functions? Which of these functions are even functions?
(6) Find the domain of the function $f(x) = \frac{\sqrt{x - 3} + \sqrt{24 - 3x}}{\sqrt{x - 4}}$.
(7) Assume $h(x) = \sqrt{4 + \cos x}$. Find functions $f$ and $g$ such that $h = g \circ f$. Find a different choice of $f$ and $g$ such that $h = g \circ f$.
(8) Give an explicit formula for a function $f$ which has all of the following properties simultaneously:
\[
\lim_{x \to 0^-} f(x) = 2, \quad f(0) = 3, \quad \lim_{x \to 0^+} f(x) \text{ does not exist}, \quad \lim_{x \to 0^+} f(x) \neq \infty, \quad \lim_{x \to 0^+} f(x) \neq -\infty.
\]
(9) Evaluate $\lim_{x \to 0^+} \frac{3 - x}{\sqrt{3x^2 - 2} - \sqrt{7}}$.
(10) Evaluate $\lim_{x \to 2} \frac{x^2 + x - 6}{3x^2 + x - 14}$.
(11) Evaluate $\lim_{x \to 0} \frac{\tan^2 x}{x^2}$.
(12) Evaluate $\lim_{x \to 0} \frac{x}{\tan(7x)}$.
(13) Evaluate $\lim_{x \to 0} \frac{-1 + \sec x}{x^2}$.
(14) Explain how the Squeeze Rule can be used to find $\lim_{x \to 0} x^4 \sin(1/x)$.
(15) Let $C$ be a constant. Let $f(x)$ be given by
\[
f(x) = \begin{cases} 
(x^3 - 8)/(2 - x) & \text{if } x \neq 2, \\
C & \text{if } x = 2.
\end{cases}
\]
Find the value of $C$ that makes $f$ a continuous function on $(-\infty, \infty)$. 

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(16) Let $A$ and $B$ be constants. Let $f(x)$ be given by

$$f(x) = \begin{cases} 
Ax^2 + B & \text{if } x \leq 1, \\
2Ax - B & \text{if } 1 < x < 2, \\
A + Bx + 1 & \text{if } 2 \leq x.
\end{cases}$$

Find the values of $A$ and $B$ that make $f$ a continuous function on $(-\infty, \infty)$.

(17) Suppose that $f$ is a continuous function on the interval $[0, 3]$. Assume $f(0) = 1$, $f(1) = 7$, $f(2) = 2$, $f(3) = 6$. Explain why the equation $f(x) = 5$ must have at least 3 solutions.

(18) Show that the equation $x^3 + 5x + 1 = 0$ must have a solution in the interval $(-10, 0)$.

(19) Find the values of $A$ and $B$ that make $f$ a continuous function on $(-\infty, \infty)$.

(20) Suppose that $f$ is a continuous function on the interval $[0, 3]$. Assume $f(0) = 1$, $f(1) = 7$, $f(2) = 2$, $f(3) = 6$. Explain why the equation $f(x) = 5$ must have at least 3 solutions.

(21) Find the annual interest rate than will double a bank deposit in 8 years (when interest is compounded continuously).

(22) Assume $f(x) = |x + 5|$. Use one-sided limits to show that $f'(-5)$ does not exist.

(23) Assume $f(x) = 3x^2 + x + 5$. Find $f'(x)$ by computing the limit of the difference quotients.

(24) Find an equation for the tangent to the curve $y = 1/x$ at the point $(3, 1/3)$.

(25) Let $f$ and $g$ be differentiable functions such that $f(3) = 4$, $f'(3) = 7$, $g(3) = 2$, $g'(3) = 6$. Find $h'(3)$, where $h(x) = f(x)/g(x)$.

(26) Let $f$, $g$ $h$ be differentiable functions such that $f(1) = 2$, $f'(1) = 5$, $g(1) = -1$, $g'(1) = 6$, $h(1) = 7$, $h'(1) = 3$. Find $u'(1)$, where $u(x) = f(x)g(x)h(x)$.

(27) Find $\frac{d}{dx}[(4 + (1/\sqrt{x}))(5x^8 + 4)]$.

(28) Find $\frac{d}{dx}[(x + \sqrt{x})(\sec x)]$.

(29) Find $\frac{d}{dx}[(\ln x)/(e^x)]$.

(30) Find $\frac{d}{dx}[(\tan x)(\sin x)(e^x)]$.

(31) Find $f''(x)$ when $f(x) = x \cos x$.

(32) A golfer hits a golf ball on a level golf course. The ball leaves the ground at time $t_1$ and hits the fairway at time $t_2$, where time is measured in seconds. The height of the ball is $-16t^2 + 64t - 32$ feet at time $t$ (where $t_1 \leq t \leq t_2$). Find $t_1$ and $t_2$. How much time did the ball spend in the air? What was the greatest height reached by the ball?

(33) Find the average rate of change of $y = x^2 + 2x + 5$ from $x = 2$ to $x = 4$.

(34) Find the instantaneous rate of change of $y = x^2 + 2x + 5$ at $x = 2$.

(35) Find the relative rate of change of $y = x^2 + 2x + 5$ at $x = 2$. 

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