# Problems for the Televised Review for Math 135
## With Answers

Answers are provided to most problems. To see the complete solution, tune in to the TV broadcast.

1. Compute the following limits. Justify you answers without resorting to calculator computations, graphing, or the use of l'Hôpital's rule.

(a) $\lim_{x\to 3} \frac{x-3}{x^2-9} = 1/6$ (b) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = 4$ (c) $\lim_{x\to 0} \frac{\tan 3x}{\tan 4x} = 3/4$

(d) $\lim_{x\to 1^+} \frac{|x^2-1|}{x-1} = 2$ (e) $\lim_{x\to 1^-} \frac{|x^2-1|}{x-1} = -2$

2. Compute the following limits. Justify your answers without resorting to calculator computations or graphing.

(a) $\lim_{x\to\infty} \frac{100x^2+x-2}{x^3-x+3} = 0$ (b) $\lim_{x\to\infty} \frac{e^x}{x^2} = \infty$ (c) $\lim_{x\to 1} \frac{\ln x}{x-1} = 1$

(d) $\lim_{x\to -\infty} \frac{2x^3-x+7}{3x^3+x^2-5} = 2/3$

3. Compute the derivatives of the following functions **from the definition**.

(a) $x^3$ (b) $\frac{2}{x-1}$ (c) $x^{1/2}$

4. Compute the derivatives with respect to $x$ of the following:

(a) $(e^x \sin x)' = e^x \sin x + e^x \cos x$ (b) $\left(\frac{\sin x}{\ln x}\right)' = \frac{\ln(x)\cos(x) - \frac{\sin(x)}{x}}{\ln^2(x)}$

(c) $(\tan(3x^2-1))' = 6x\sec^2(3x^2-1)$ (d) $\left(\int_0^{x^2} \sin t\, dt\right)' = 2x\sin(x^2)$

(e) $\left(\int_{\sin x}^{\cos x} e^t\, dt\right)' = -\sin(x)e^{\cos(x)} - \cos(x)e^{sin(x)}$

5. Compute the following indefinite integrals:

(a) $\int (x^4 - x^{-1} + \sin x - 2e^x)\, dx = \frac{x^5}{5} - \ln(|x|) - \cos x - 2e^x + C$

(b) $\int x^2 \sin(x^3)\, dx = -\frac{1}{3}\cos(x^3) + C$

(c) $\int e^x \sec^2(e^x)\, dx = \tan(e^x) + C$

6. Evaluate the Riemann sum for the funtion $x^2+x$ on the interval $[0,6]$ using the partition 0, 2, 4, 6 and taking as the representative points the midpoint of each subinterval.
Ans.: 88

7. Evaluate the following definite integrals:

(a) $\displaystyle\int_0^1 (x^3 - x^2)\, dx = -\frac{1}{12}$ (b) $\displaystyle\int_0^2 xe^{x^2}\, dx = \frac{1}{2}(e^4 - 1)$

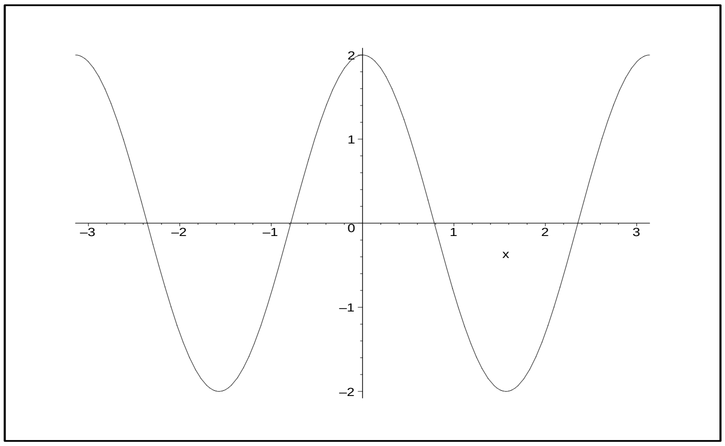
8. Find the areas of the following regions in the plane:

(a) The region bounded by the $x$-axis, the graph of $y = x^3 + 1$, and the line $x = 1$.
Ans.: 2

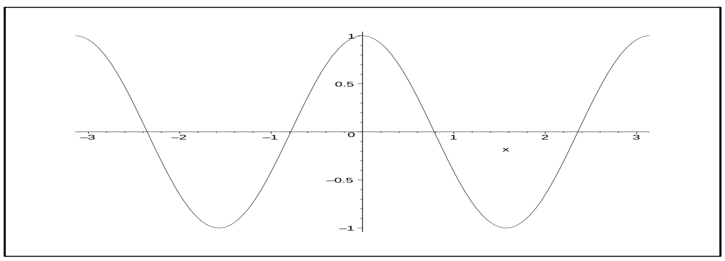
(b) The region in the first quadrant bounded by the graph of $y = 16 - x^4$.
Ans.: 128/5

9. Here are the graphs of four functions. Two of these functions are the derivatives of the other two. Match the functions with their derivatives. (When viewing the graphs with Adobe Reader, use the zoom feature to examine the graphs carefully.)

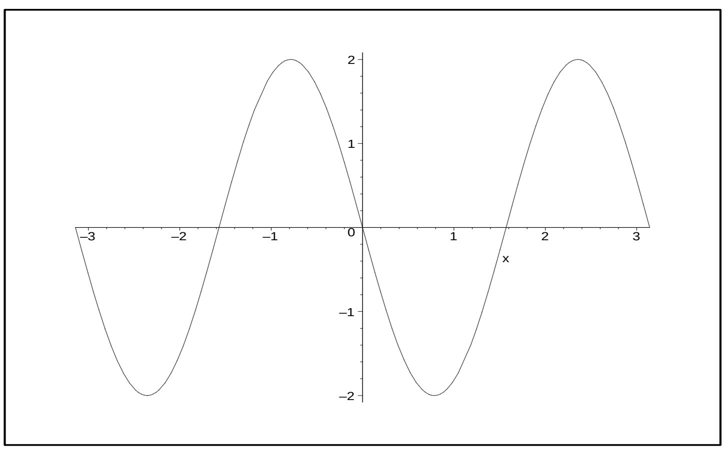
(a)



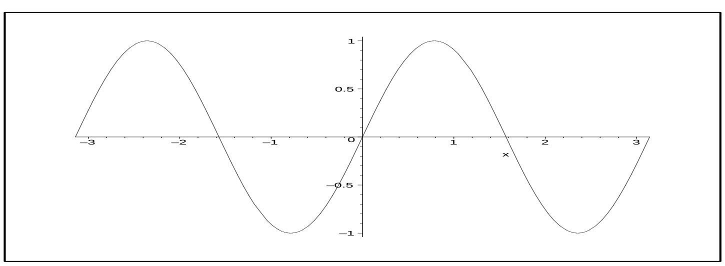

(b)



(c)




(d)



(a) is the derivative of (d) and (c) is the derivative of (b).

10. Draw graphs of functions with the following properties:

(a) A function $f$ such that $f(x)$ is defined and differentiable for all $x$ except $x = -2$ and $x = 1$,

$$\lim_{x\to\infty} f(x) = 2 \quad \text{and} \quad \lim_{x\to-\infty} f(x) = -1,$$

the graph of $f$ has vertical asymptotes at $x = -2$ and $x = 1$, $f(x)$ is never equal to 0, and $f(0) = -3$.

There are many possible graphs.

(b) A function $g$ such that the domain of $g$ is $(-4, 4)$, $g(x)$ is differentiable at all points $x$ in its domain, $g$ has critical numbers $x = -2$ and $x = 1$, $g'(0) = -1$, $g$ is concave downward on $(-4, 0)$ and concave upward on $(0, 4)$.

There are many possible graphs.

11. Find the minimum and maximum values of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 7$ on the interval $[-2, 3]$.

The minimum value is $-25$ and the maximum value is 39.

12. Assume that $u$ and $v$ are differentiable functions defined for all real numbers and that $f(x) = u(v(x))$. Suppose that the following values of $u$, $v$, and their first and second derivatives are known:

| $x$ | $u(x)$ | $v(x)$ | $u'(x)$ | $v'(x)$ | $u''(x)$ | $v''(x)$ |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | $-1$ | 1 | 2 | 3 |
| 2 | 4 | 0 | 5 | $-2$ | ? | ? |

(a) What is $f'(1)$? Ans.: 5

(b) If $f''(1) = 7$, then what is $u''(2)$? Ans.: $-8$

13. Assume that $p$ and $q$ are differentiable functions defined for all real numbers and that $g(x) = p(x)q(x)$. Suppose that the following values of $p$, $q$, and their first and second derivatives are known:

| $x$ | $p(x)$ | $q(x)$ | $p'(x)$ | $q'(x)$ | $p''(x)$ | $q''(x)$ |
|---|---|---|---|---|---|---|
| 2 | 2 | $-3$ | $-1$ | 1 | ? | 7 |
| 3 | 1 | $-1$ | 3 | ? | 4 | 2 |

(a) What is $g'(2)$? Ans.: 5

(b) If $g''(3) = 5$, what is $q'(3)$? Ans.: 7/6

14. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/sec. At what rate is his distance to second base decreasing when he is halfway to first base?

Ans.: Decreasing at $\frac{24\sqrt{5}}{5}$ feet per second.

15. In the following problem distances along the axes are measured in centimeters. A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4, 2)$, its $x$-coordinate is increasing at the rate of 3 cm/sec. How fast is the distance of the particle from the origin changing at this instant?

Ans.: Increasing at $\frac{27\sqrt{5}}{20}$ centimeters per second.

16. A piece of wire 10 m long is cut into two pieces. On piece is bent into a square and the other into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum? (Note: An equilateral triangle with side $s$ has area $s^2\sqrt{3}/2$.)

Ans. The formula given for the area of the triangle is wrong! The correct formula is $s^2\sqrt{3}/4$. If one uses the formula as stated, one get that a maximum occurs when all of the wire is used for the triangle and a minimum occurs when $\dfrac{80\sqrt{3}}{9+8\sqrt{3}}$ meters is used for the square and the rest for the triangle.

However, if one uses the correct formula, one finds that a maximum occurs when all of the wire is used for the square and a minimum occurs when $\dfrac{40\sqrt{3}}{9+4\sqrt{3}}$ meters is used for the square and the rest is used for the triangle.

17. Choose real numbers $a$ and $b$ such that the polynomial $f(x) = 2x^3 + ax^2 + bx + 7$ has a relative maximum of 27 at $x = -2$. What other relative extrema does $f$ have? Where is the point of inflection for the graph of $f$?

Ans.: $a = 3$, $b = -12$. There is a relative minimum at $x = 1$ and a point of inflection at $x = -1/2$.

18. Let $f(x) = \dfrac{x}{2+3x^2}$. Describe the interval(s) on which $f$ is increasing and the interval(s) on which $f$ is decreasing. Does the graph of $f$ have any vertical or horizontal asymptotes? If so, what are they? Does $f$ have any absolute extrema? If so, where do they occur?

Ans.: $f$ is increasing on the interval $(-\sqrt{6}/3, \sqrt{6}/3)$ and $f$ is decreasing on the intervals $(-\infty, -\sqrt{6}/3)$ and $(\sqrt{6}/3, \infty)$. The $x$-axis is a horizontal asymptote. $f$ has an absolute minimum at $x = -\sqrt{6}/3$ and an absolute maximum at $x = \sqrt{6}/3$.

19. Let $g(x) = e^{2x-x^2}$. Describe the interval(s) on which $g$ is increasing and the interval(s) on which $g$ is decreasing? Describe the interval(s) on which the graph of $g$ is concave up and the interval(s) on which the graph is concave down. Does the graph of $g$ have

any vertical or horizontal asymptotes? If so, what are they? Does $g$ have any absolute extrema? If so, where do they occur?

Ans.: $g$ is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$. The graph of $g$ is concave up on $(-\infty, 1-\sqrt{2}/2)$ and $(1+\sqrt{2}/2, \infty)$ and the graph is concave down on $(1-\sqrt{2}/2, 1+\sqrt{2}/2)$. The $x$-axis is a horizontal asymptote. $g$ has an absolute maximum at $x = 1$.

20. Let $h(x) = \dfrac{(x^3-1)^{1/3}}{|x-2|}$. Find equations for all the horizontal and vertical asymptotes of $h$.

Ans.: The lines $y = 1$ and $y = -1$ are horizontal asymptotes and the line $x = 2$ is a vertical asymptote.

21. Let $f(x) = (1 + 7x^2)^{1/3}$. Use the linearization of $f$ at $x = 1$ to estimate $f(1.07)$.

Ans.: $2 + 7(.07)/6 = 2.08166666\ldots$

22. Find the largest possible value of $x^2y$ when $x$ and $y$ are subject to the conditions $x \geq 0$, $y \geq 0$, $2x + 3y = 1$.

Ans.: $1/81$

23. A rectangular poster is to have 192 square inches of printed matter with 4 inch margins at the top and bottom and 3 inch margins on the sides. What should the overall dimensions of the poster be in order to minimize the amount of paper needed?

Ans.: 18in by 24in

24. Find the slope of the tangent line to the graph of the equation $x^4y^5 + xy = 18$ at the point $(2, 1)$.

Ans.: $-33/82$

25. Suppose that $f$ is a function such that $f''(x) = \sin(2x)$, $f(0) = 2$, and $f'(0) = 3$.

(a) Find $f'(x)$. (b) Find $f(x)$.

Ans.: (a) $7/2 - \cos(2x)/2$ (b) $2 + 7x/2 - \sin(2x)/4$

26. A particle is moving along the $x$-axis in such a way that its position at time $t$ is

$$t^4 - 12t^3 + 30t^2 + 32t + 10$$

for $0 \leq t \leq 6$. At what time is the speed, the absolute value of the velocity, a maximum?

Ans.: $t = 5$

27. A particle is moving along the $x$-axis in such a way that its position at time $t$ is

$$100 + 20t + 66t^2 + 20t^3 - t^4$$

for $0 \leq t \leq 10$. At what time is the particle undergoing maximum acceleration?

Ans.: $t = 5$