135 first exam formula sheet. **Any addition will void the use of this sheet!**

Definition: $\lim_{x\to a} f(x) = L$ if the value $f(x)$ can be made as close to the number L as we please by taking x sufficiently close to (but not equal to) a.
Theorem : $\lim_{x\to a} f(x) = L$ iff $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$.
This theorem says that $\lim_{x\to a} f(x)$ exists iff :
1. $\lim_{x\to a^+} f(x)$ exists *and* 2. $\lim_{x\to a^-} f(x)$ exists *and* 3. Both limits in 1. and 2. are equal.
Definition: f is **continuous** at a if the following conditions are satisfied :
1. $f(a)$ is defined.
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

The Intermediate Value Theorem :
If f is a continuous function on a closed interval $[a, \ b]$ and M is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, \ b]$ such that $f(c) = M$.

Definitions: $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ *so* $f'(a) = f'(x)|_{x=a} = \lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

f is **differentiable** at a if $f'(a)$ exists.

Fact: $f'(a)$ is the slope of the tangent line to the graph of f at $x = a$

Differentiation rules:
Product rule: $[f(x) * g(x)]' = f'(x) * g(x) + g'(x) * f(x)$

Quotient rule: $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{g(x) * f'(x) - f(x) * g'(x)}{g^2(x)}$

Power Rule: $(x^r)' = rx^{r-1}$

Chain Rule: If $h(x) = g[f(x)]$, then $h'(x) = g'(f(x)) * f'(x)$

Equivalently, if we write $y = h(x) = g(u)$, where $u = f(x)$, then $\dfrac{dy}{dx} = \dfrac{dy}{du} * \dfrac{du}{dx}$

Calculus in economics definitions, from section 3.4:
Here p is the price per unit and x the number of units. The demand equation relates p and x and can be solved for p as a function of x, or x as a function of p.
Revenue $R = px$. (Here usually p is written as a function of x, using the demand equation, so that R becomes a function of x only.)
Profit P equals revenue R minus total cost C.
Average cost $\bar{C}(x) = \frac{C(x)}{x}$
Elasticity of demand: if f is a differentiable demand function written as $x = f(p)$, then the elasticity of demand at price p is given by $E(p) = -\dfrac{pf'(p)}{f(p)}$. The demand is **elastic** (price increase causes revenue to decrease) if $E(p) > 1$, **inelastic** (price increase causes revenue to increase) if $E(p) < 1$, and **unitary** if $E(p) = 1$.

Geometry: For a sphere of radius r, the volume is $\frac{4}{3}\pi r^3$ and surface area is $4\pi r^2$.