

Formula Sheet for Exam II in Calculus Spring 2002

If $y = f(x)$, the **differential** dy is $dy = f'(x)dx$. Compare with $\Delta y = f(a + \Delta x) - f(a)$.
 $f(a + \Delta x) = f(a) + \Delta y \approx f(a) + dy = f(a) + f'(a)dx$

The Mean Value Theorem : If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , then there is a $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

Asymptotes: The line $x = a$ is a **vertical asymptote** of the graph of a function f if $\lim_{x \rightarrow a^+} f(x) = \infty$ or $-\infty$ or $\lim_{x \rightarrow a^-} f(x) = \infty$ or $-\infty$. In particular, if $f(x) = \frac{P(x)}{Q(x)}$ when P, Q are polynomial functions, then the line $x = a$ is a vertical asymptote of f if $Q(a) = 0$ but $P(a) \neq 0$.

The line $y = b$ is an **horizontal asymptote** of the graph of a function f if $\lim_{x \rightarrow \infty} f(x) = b$ or $\lim_{x \rightarrow (-\infty)} f(x) = b$

A **critical point** of a function f is any point x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist.

For **absolute extrema** of a continuous function on a closed interval $[a, b]$:

Compare the value of the function for all the critical points in the interval (a, b) and for the two endpoints $x = a$ and $x = b$.

Local extremum tests:

First derivative test at a critical point c of f : if sign of f' changes at c from - to + as x increases, then f has a local min at c ; if sign of f' changes from + to -, then f has a local max at c .

Second derivative test at a critical point c of f : if $f''(c) > 0$, then f is concave up at c and has a local min there; if $f''(c) < 0$, then f is concave down at c and has a local max there.

If $(x, f(x))$ is an **inflection point** of a function f then x is in the domain of f and the concavity changes at $(x, f(x))$ (which requires either $f''(x) = 0$ or $f''(x)$ does not exist).

Basic log and exp laws: $e^{\ln x} = x$ for $x > 0$ $\ln e^x = x$ for all real numbers x
 $\ln(mn) = \ln m + \ln n$ $\ln(m/n) = \ln m - \ln n$ $\ln(m^n) = n \ln m$ $\ln 1 = 0$ $\ln e = 1$
 $e^a e^b = e^{(a+b)}$ $\frac{e^a}{e^b} = e^{(a-b)}$ $(e^a)^b = e^{(ba)}$ $1 = e^0$ $e = e^1$

Basic trig identities: $\sin^2 \theta + \cos^2 \theta = 1$ $\sin(-x) = -\sin x$ $\cos(-x) = \cos x$
 $\sin(2\pi + x) = \sin x$ $\cos(2\pi + x) = \cos x$ $360 \text{ degrees} = 2\pi \text{ radians}$
 $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\csc x = \frac{1}{\sin x}$

Basic differentiation rules plus chain rule (both forms) for trig, log, and exp functions:

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}[\sin(f(x))] = [\cos(f(x))]f'(x)$	$\frac{d}{dx}(\sin u) = (\cos u)\frac{du}{dx}$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}[\cos(f(x))] = -[\sin(f(x))]f'(x)$	$\frac{d}{dx}(\cos u) = -(\sin u)\frac{du}{dx}$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}[\tan(f(x))] = [\sec^2(f(x))]f'(x)$	$\frac{d}{dx}(\tan u) = (\sec^2 u)\frac{du}{dx}$
$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(e^{f(x)}) = [e^{f(x)}]f'(x)$	$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$
$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x \neq 0)$	$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)} \quad (f(x) > 0)$	$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx} \quad (u \neq 0)$