Review Problems for First Midterm Exam in Calculus 135 Spring 2002

NOTE: These are only practice problems!
The number of problems in the exam will be less than this review.
You are responsible to study **all** the material and should be able to do **all** homework problems!
You can find additional problems in the 135 Fall 2000 webpage.

1. Let $f(x) = \sqrt{x^2 + 3x - 40}$, $\quad g(x) = \frac{1}{x}$
a. Find $g\big(f(x)\big) \quad f\big(g(x)\big)$. $\quad$ b. Find the domain of $g\big(f(x)\big)$.

2. Find the value of each of the following limits in 3 different ways: analytically (exactly), graphically, (illustrating the limit on a graph) and numerically (give numerical evidence for the limit).

a. $\lim\limits_{x\to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4}$ $\qquad$ b. $\lim\limits_{x\to 4} \dfrac{x^2 - 5x + 6}{x^2 - 4}$ $\qquad$ c. $\lim\limits_{x\to\infty} \dfrac{3x^4 + 10x + 7}{2x^4 - 5x^3}$

3. Let $g(x) = x^3 + x + 100$.**Without graphing** the function g, explain why there is at least one number $c \in (-5, 1)$ such that $g(c) = 0$. **Don't try to find c!**

4. The tangent line to the curve of some unknown function $g(x)$ at $x = 3$ is $2y + x = 6$. What is $g(3)$? What is $g'(3)$?

5. Let

$$f(x) = \begin{cases} x^3 + 2 & if \quad x > 1 \\ C & if \quad x = 1 \\ 5x - 2 & if \quad 1 > x \geq 0 \\ x^2 - 2 & if \quad x < 0 \end{cases}$$

a. For what value of C is f continuous at $x = 1$? **Explain !**
b. Is f continuous at $x = 0$ **Explain!**
c. Find the following limits: $\lim\limits_{x\to 2} f(x)$, $\quad \lim\limits_{x\to -1} f(x)$
d. Is $f(x)$ differentiable at $x = 0$? **Explain!**
e. Check your answers to b) and d) by sketching the graph of $f(x)$ on the interval $[-1, 1)$.

6. Use the definition of the derivative as a limit to find the derivative of $f(x) = x^2 + 2$

7. Sketch a possible graph of F on $[-4, 4]$ such that:
F is continuous on $[-4, -1)$and $(-1, 4]$, $\lim\limits_{x\to -1^-} F(x) = 5$, $\lim\limits_{x\to -1^+} F(x) = 2$, F is not differentiable only at $x = -1$ and $x = 1$.

8. Find the equation of the tangent line to the curve given by the **implicit** function
$y^2 - xy = x^2 + y - 6$ at $(2, 1)$.

9. Let $h(x) = f\big(g(x)\big)$. Assume that $f'(2) = 5$, $\quad g(1) = 2$, and $g'(1) = 12$. What is $h'(1)$?

10. Find the derivative of the following functions. Don't simplify!
a. $f(x) = \sqrt[5]{x^3} + \frac{6}{x^{\frac{3}{8}}} + x^3 + 7$ $\qquad$ b. $g(x) = (x + 3x^9)(x^5 - 7x)$

c. $h(x) = \left(\dfrac{x^2 + 3x}{x^6 - 9x}\right)^5$ $\qquad$ d. $k(x) = \sqrt[3]{x^7 + 5x^2 - 50}$

11. a. Find the second derivative of the function $F(x) = 5\sqrt{x}$

b. Find $\dfrac{d^9 g}{dt^9}$ where $g(t) = 120t^7 + t^6 - 62t^4 + 930$.

12. Suppose that the total monthly cost function (in dollars) associated with manufacturing x Starview brand telescopes is given by $C(x) = 6,000 + 4x$ when $0 \leq x \leq 10,000$. The wholesale **unit** price is $p = 120 - 0.001x$ where x denotes the monthly quantity demanded.

Recall that the revenue is given by xp and the profit by $P(x) = R(x) - C(x)$.

a. Find the daily revenue function $R(x)$.

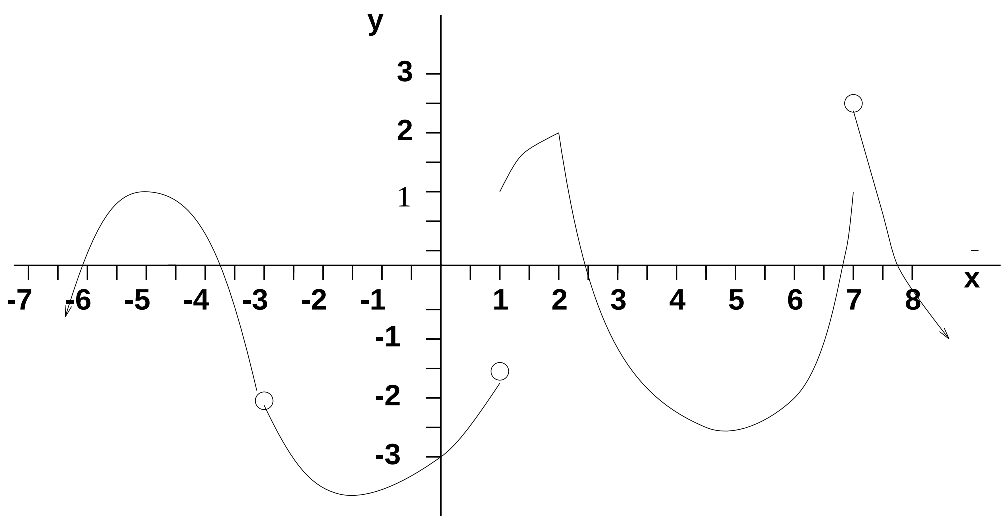
b. Find the daily profit function $P(x)$.

c. Find the daily marginal profit function.

d. Find the approximate actual profit realized from the sale of the $4,001^{st}$ telescope.

e. Determine whether the demand is elastic, unitary, or inelastic when $p = 30$, and when $p = 90$.

13. Tom is blowing air into a soap bubble at the rate of $10\text{cm}^3/\text{sec}$. Assuming that the bubble is spherical, how fast is its radius changing when the radius is 5cm? How fast is the surface area of the bubble changing at that instant of time?



Graph used in problem 14

14. Use the graph of F (above) to answer a, b, and c for $x = a$ where $a = -3, 0, 1, 2, 6, 7$.

a. What is $\lim_{x \to a} F(x)$? If the limit does not exist, write DNE and explain why.

b. Is F continuous at $x = a$? If F is discontinuous at $x + a$, explain why.

c. Is F differentiable at $x = a$? If F is not differentiable at $x = a$, explain why.