

Review Problems for Second Midterm Exam in Calculus 135 Spring 2002

NOTE: These are only practice problems!

The number of problems in the exam will be less than this review. You are responsible for studying **all** the material and should be able to do **all** homework problems!

1. Sketch a graph of a continuous and differentiable function $f(x)$ with the following properties: The only critical points of f are $x = 0$, $x = 2$, and $x = 6$. Further, $f''(x) = 0$ only when $x = 1$ and $x = 4$. Finally, $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = -\infty$.

2. Let $f(x) = 2x^3 - 6x^2 - 18x + 3$.

(a). Find all relative extrema of $f(x)$. Apply the second derivative test or the first derivative test to determine which are relative minima and which are relative maxima.

(b). Find the absolute maximum and absolute minimum of $f(x)$ on $[-2, 2]$.

3. An open box with a square base and a total volume of 100 cubic inches is to be constructed from two types of materials. The base should be made of a heavy duty metal which costs \$4 per square inch and the sides should be made of cardboard which cost 80 cents per square inch. Find the dimensions of the box that will minimize the total cost.

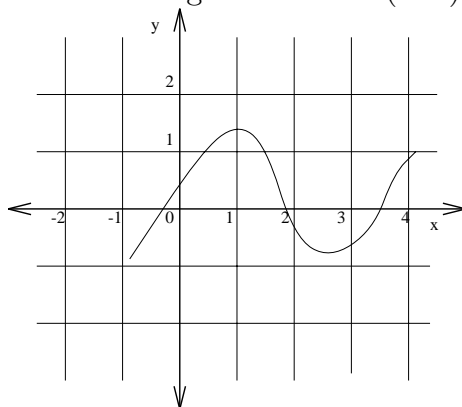
Don't forget to check that your answer **minimizes** the cost.

4. Sketch the graph of the function $f(x) = \frac{x^2-1}{x^2-4}$ using **calculus only!** (It is OK to use your calculator to check your answer, but you need to say, based on calculus, how the answers are obtained.)

Show all work: find the domain of $f(x)$, and find $f(0)$, and all vertical and horizontal asymptotes of $f(x)$ **with the appropriate limits**.

Find where the function is increasing/decreasing and where it is concave up and down. Indicate on the graph all local extrema and all inflection points, if any.

5. Use the graph of the function $G(x)$ below, to determine if $G(x)$, $G'(x)$, and $G''(x)$ are positive, negative, or zero at the following values of x : (-0.5) , 0 , 0.5 , 1 , 2 , 2.5 , and 3 .



Graph of $G(x)$

6. Use logarithmic differentiation to compute the derivative of $\frac{(x^5 + 6x^2)^4(x^6 + x^4 + 12)}{(x^2 + 5)^2x^8}$

7. The monthly demand for PCs is given by $D(t) = 2000 - 1500e^{-0.05t}$. (a). When will the demand D be 1000 PCs/month? (b). Find the rate of growth of demand when $D = 1000$.

8. The **derivative** of f is $f'(x) = x^{15}(x + 1)^{10}$.

(a). Find all the critical points of f and check where f is increasing and decreasing.

(b). Compute $f''(x)$ and determine the x values at the points of inflection.

Note: **Do no try to find f !**

9. Find the derivatives of the following functions:

$$F(x) = \tan^2 x + e^x \sin 8x \quad G(x) = \ln \frac{x^2+5}{x+6} + e^{x^3} \quad H(x) = \cos x^2 + 5 + x^2 e^x + \frac{1}{e}$$

10. Find the absolute maximum and minimum of the function $g(x) = \sin x + \cos x$ on $[0, 2\pi]$.

11. Use differentials to approximate the values of $\sqrt[3]{8.01}$ and $\sqrt[3]{7.99}$.

12. The weekly quantity demanded of “Comfy” chairs is given by $p+0.2x^2 = 280$ where p is measured in dollars and x is the number of chairs demanded per week. (a). Use differentials to estimate the change in the price of a unit when the weekly quantity demanded changes from 20 to 21. (b). Determine the elasticity when $x = 20$ and $p = 200$.

13. (a). Sketch the graph of $F(x) = x^2 + 1$ on $[0, 4]$.

(b). Show the Mean Value Theorem graphically (on your graph only) when $a = 0$ and $b = 4$.

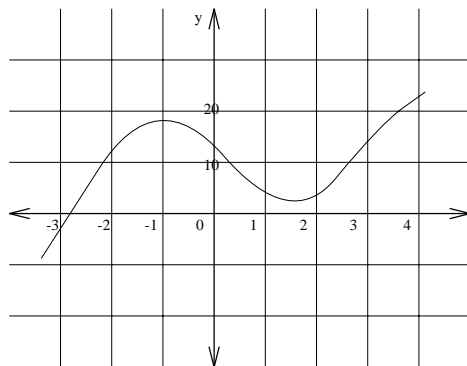
14. Below is the graph of the derivative $f'(x)$ of a differentiable function f . Note that it is NOT the graph of f itself!

(a). Find all the critical points of f (NOT f'), and for each check whether the function f has a relative maximum or a relative minimum.

(b). Find where the function f is increasing and where it is decreasing.

(c). Find where the function f is concave up or down. Determine the location of the inflection points.

(d). Use the information from a, b, and c above to sketch the graph of a possible function f (so that the graph below is indeed the graph of its derivative).



Graph of $f'(x)$