Formula sheet for the final exam in Math 135

Preliminaries: The line through the point (h,k) with slope m is given by the equation $y-k=m(x-h)$. If $a\neq 0$ and $ax^2+bx+c=0$ then $x=(-b\pm\sqrt{b^2-4ac})/(2a)$.

Basic trig identities: $\sin(0)=0$, $\cos(0)=1$, $\sin^2 x+\cos^2 x=1$, $\sin(-x)=-\sin x$, $\cos(-x)=\cos x$

$$\sin(x+2\pi)=\sin x \qquad \cos(x+2\pi)=\cos x \qquad 360 \text{ degrees} = 2\pi \text{ radians}$$

$$\tan x=(\sin x)/(\cos x) \qquad \cot x=(\cos x)/(\sin x) \qquad \sec x=1/(\cos x) \qquad \csc x=1(/\sin x)$$

Limits: $\lim\limits_{x\to a} f(x)=L$ is equivalent to $\lim\limits_{x\to a^+} f(x)=\lim\limits_{x\to a^-} f(x)=L$.

Continuity: f is continuous at a when all of the following conditions are satisfied:
1. $f(a)$ is defined, 2. $\lim\limits_{x\to a} f(x)$ exists, 3. $\lim\limits_{x\to a} f(x)=f(a)$.

Derivatives: f is differentiable at x when $f'(x)=\lim\limits_{h\to 0}\big(f(x+h)-f(x)\big)/h$ exists. The number $f'(a)$ is the slope of the tangent to the graph of $y=f(x)$ at $x=a$. The **sum and product rules** say $(f+g)'=f'+g'$ and $(fg)'=fg'+gf'$, respectively. The **quotient rule** says $(f/g)'=(gf'-fg')/(g^2)$. The **chain rule** says $\dfrac{d}{dx}[f(g(x))]=f'(g(x))\dfrac{d}{dx}[g(x)]$. $f'(c)/f(c)$ is the relative rate of change of $y=f(x)$ at $x=c$. $f'(c)$ is the instantaneous rate of change. If $s(t)$ is the position function then $s'(t)$ is the velocity, $s''(t)$ is the acceleration and $|s'(t)|$ is the speed.

Basic log and exp laws: $e^{\ln x}=x$ for $x>0$ $\quad \ln(e^x)=x$ for all x

$$\ln(ab)=\ln a+\ln b \qquad \ln(a/b)=\ln a-\ln b \qquad \ln(a^b)=b\ln a \qquad \ln 1=0 \qquad \ln e=1$$

$$e^a e^b=e^{a+b} \qquad e^a/e^b=e^{a-b} \qquad (e^a)^b=e^{ab} \qquad e^0=1 \qquad e^1=e$$

Exponential growth is given by the formula $P(t)=P_0e^{kt}$. **Continuous compounding** of interest leads to the formula $A(t)=Pe^{rt}$, where r is the interest rate.

Calculus in economics: $C(x)$ is the **cost** of producing x units of a product. The **marginal cost** is $C'(x)$; the **average cost** is $C(x)/x$. If x units are available then $p(x)$ is the **price** consumers are willing to pay for each unit. $R(x)=xp(x)$ is the **revenue** from x units. $R'(x)$ is the **marginal revenue**. The **profit function** is $P(x)=R(x)-C(x)$.

Approximation: The **linearization** of $f(x)$ at $x=a$ is $L(x)=f(a)+f'(a)(x-a)$. If $y=f(x)$ then its **differential** is $dy=f'(x)dx$. This differential is approximately equal to $\Delta y=f(x+\Delta x)-f(x)$ when $dx=\Delta x$. For any variable S, ΔS is the **error**, $(\Delta S)/S$ is the **relative error** and $100\big((\Delta S)/S\big)\%$ is the **percentage error**.

Graphing and optimization: If $f'(x)>0$ on an interval then f is **increasing** on that interval. If $f'(x)<0$ on an interval then f is **decreasing** on that interval. If $f''(x)>0$ on an interval then f is **concave up** on that interval. If $f''(x)<0$ on an interval then f is **concave down** on that interval. **Relative max and min** of a function f can occur only at **critical numbers** (numbers x in the domain of f where $f'(x)=0$ or $f'(x)$ does not exist). **Absolute max and min** occur only at critical points or endpoints. **Inflection points** are points where the concavity of f changes sign. The line $x=a$ is a **vertical asymptote** of $y=f(x)$ if either $\lim\limits_{x\to a^+} f(x)=\pm\infty$ or $\lim\limits_{x\to a^-} f(x)=\pm\infty$. The line $y=b$ is a **horizontal asymptote** of $y=f(x)$ if either $\lim\limits_{x\to\infty} f(x)=b$ or $\lim\limits_{x\to-\infty} f(x)=b$.

L'Hôpital's Rule: If $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}$ is a form of type $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$, then $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$ when the second limit exists.

The indefinite integral: If $F'(x) = f(x)$ then F is an **antiderivative** of f. $\int f(x)\,dx$ is the set of all antiderivatives of f. $F'(x) = f(x)$ is equivalent to $\int f(x)\,dx = F(x) + C$, where C is a constant.

The definite integral: If f is continuous and $f(x) \geq 0$ on $[a,b]$ then $\int_a^b f(x)\,dx$ is the **area** under the curve $y = f(x)$ over $[a,b]$. If $f(x) < 0$ occurs for some x then $\int_a^b f(x)\,dx$ is the area above $[a,b]$ minus the area below $[a,b]$. The **first fundamental theorem of calculus** says $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f.

Riemann sums are of the form $\sum_{k=1}^n f(x_k^*)\Delta x_k$ where $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$, $\Delta x_k = x_k - x_{k-1}$ and x_k^* is chosen from $[x_{k-1}, x_k]$. They approximate $\int_a^b f(x)\,dx$.

Differentiation rules	**Integration rules**
$\frac{d}{dx}(kx) = k$ for constant k	$\int k\,dx = kx + C$ for constant k
$\frac{d}{dx}[kf(x)] = kf'(x)$ for constant k	$\int kf(x)\,dx = k\int f(x)\,dx$ for constant k
$\frac{d}{dx}(x^k) = kx^{k-1}$ for constant k	$\int x^k\,dx = \frac{x^{k+1}}{k+1} + C$ for constant $k \neq -1$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$

Integration by substitution: If $u = u(x)$ then $\int f(u(x))u'(x)\,dx = \int f(u)\,du$ and $\int_a^b f(u(x))u'(x)\,dx = \int_{u(a)}^{u(b)} f(u)\,du$.