Formula Sheet for Exam 1

Preliminaries. $(x-a)^2+(y-b)^2=r^2$ is an equation of a circle with center (a,b) and radius r. If $x>0$ then $|x|=x$. If $x<0$ then $|x|=-x$. The inequality $|x|\le a$ is equivalent to $-a\le x\le a$. Similarly, the inequality $|x|<a$ is equivalent to $-a<x<a$. A short table of exact trig values is given below:

$$\cos(0)=1,\ \cos(\pi/6)=\sqrt{3}/2,\ \cos(\pi/4)=\sqrt{2}/2,\ \cos(\pi/3)=1/2,\ \cos(\pi/2)=0,$$
$$\sin(0)=0,\ \sin(\pi/6)=1/2,\ \sin(\pi/4)=\sqrt{2}/2,\ \sin(\pi/3)=\sqrt{3}/2,\ \sin(\pi/2)=1.$$

This short table can be expanded through the use of the basic trig identities

$$\tan x=(\sin x)/(\cos x),\ \cot x=(\cos x)/(\sin x),\ \sec x=1/(\cos x),\ \csc x=1/(\sin x),$$
$$\cos(-x)=\cos(x),\ \cos(\pi-x)=-\cos(x),\ \cos(x+2\pi)=\cos(x),$$
$$\sin(-x)=-\sin(x),\ \sin(\pi-x)=\sin(x),\ \sin(x+2\pi)=\sin(x).$$

For example, $\cos(5\pi/6)=\cos(\pi-\pi/6)=-\cos(\pi/6)=-\sqrt{3}/2$. The identities $\cos^2 x+\sin^2 x=1$ and $1+\tan^2 x=\sec^2 x$ are also useful.

The line through (x_1,y_1) and (x_2,y_2) has slope $m=(y_2-y_1)/(x_2-x_1)$ when $x_1\ne x_2$. The line through (h,k) with slope m is given by the equation $y-k=m(x-h)$. If two perpendicular lines have slopes m_1 and m_2, then we must have $m_2=-1/m_1$. The line $y=mx+b$ has slope m and y-intercept b.

If $a\ne 0$ and $ax^2+bx+c=0$ then $x=(-b\pm\sqrt{b^2-4ac})/(2a)$. We often use the factorizations $x^2-a^2=(x-a)(x+a)$ and $x^3-a^3=(x-a)(x^2+ax+a^2)$.

Limits. $\lim\limits_{x\to c^-} f(x)$ is the limit of $f(x)$ as x approaches c from the *left* (which is the case $x<c$). $\lim\limits_{x\to c^+} f(x)$ is the limit of $f(x)$ as x approaches c from the *right* (which is the case $x>c$). If $\lim\limits_{x\to c^-} f(x)=L$ and $\lim\limits_{x\to c^+} f(x)=L$ (same L), then $\lim\limits_{x\to c} f(x)=L$. If $\lim\limits_{x\to c^-} f(x)\ne\lim\limits_{x\to c^+} f(x)$ then $\lim\limits_{x\to c} f(x)$ does not exist. One can show $\lim_{x\to 0}\big((\sin x)/x\big)=1$ and $\lim_{x\to 0}\big((\cos x-1)/x\big)=0$.

Continuity. The function f is continuous at c when all three of the following conditions are fulfilled: $f(c)$ is defined, $\lim_{x\to c} f(x)$ exists, $\lim_{x\to c} f(x)=f(c)$.

Log and exp. In addition to $b^{\log_b x}=x$, $\log_b(b^x)=x$ and $\ln x=\log_e x$ we have

$$b^x b^y=b^{x+y},\ (b^x)/(b^y)=b^{x-y},\ (b^x)^y=b^{xy},\ (ab)^x=a^x b^x,\ (a/b)^x=(a^x)/(b^x),\ b^0=1,\ b^1=b,$$
$$\log_b(xy)=\log_b x+\log_b y,\ \log_b(x/y)=\log_b x-\log_b y,\ \log_b(x^p)=p\log_b x,\ \log_b(1)=0,\ \log_b(b)=1.$$

Exponential growth is given by the formula $P(t)=P_0e^{kt}$. Continuous compounding of interest leads to the formula $A(t)=Pe^{rt}$, where r is the interest rate.

Derivatives. If $y=f(x)$ then $\frac{dy}{dx}=\frac{d}{dx}(f(x))=f'(x)=\lim_{\Delta x\to 0}(f(x+\Delta x)-f(x))/(\Delta x)$ if the limit exists. The number $f'(c)$ is the slope of the tangent to the graph of $y=f(x)$ at $x=c$.

If c and n are constants then $\frac{d}{dx}(c)=0$, $\frac{d}{dx}(x)=1$, $\frac{d}{dx}(cf(x))=c\frac{d}{dx}(f(x))$ and $\frac{d}{dx}(x^n)=nx^{n-1}$. The sum and product rules say $(f+g)'=f'+g'$ and $(fg)'=fg'+gf'$, respectively. The quotient rule says $(f/g)'=(gf'-fg')/(g^2)$. In addition:

$$\frac{d}{dx}\sin x=\cos x,\ \frac{d}{dx}\cos x=-\sin x,\ \frac{d}{dx}\tan x=\sec^2 x,\ \frac{d}{dx}\sec x=\sec x\tan x,$$
$$\frac{d}{dx}\cot x=-\csc^2 x,\ \frac{d}{dx}\csc x=-\csc x\cot x,\ \frac{d}{dx}e^x=e^x,\ \frac{d}{dx}\ln x=\frac{1}{x}.$$

$f'(c)/f(c)$ is the relative rate of change of $y=f(x)$ at $x=c$. $f'(c)$ is the instantaneous rate of change. If $s(t)$ is the position function then $s'(t)$ is the velocity, $s''(t)$ is the acceleration and $|s'(t)|$ is the speed.