Review problems for Exam 1 in Math 135 - Spring 2004

NOTE: These are only practice problems! Your exam may have types of problems that are not represented on this sheet. It is your responsibility to study **all** of the material and to master **all** of the homework problems. The answers to the questions must be exact answers. For example, the number 1.732050808 is not a correct answer when the right answer is actually $\sqrt{3}$.

(1) (a) Solve for x in the equation $|9-x| = |x-6|$. (b) Solve the inequality $|9-x| \leq |x-6|$. Hint: Take the square of both sides.

(2) Find an equation of the circle with center $(-1, 3)$ which passes through the point $(1, 4)$

(3) Find the center and the radius of the circle $2x^2 + 2y^2 - 4x + 8y + 2 = 0$.

(4) Find an equation for the line passing through $(1, 2)$ which is perpendicular to the line $3x + 5y = 7$

(5) Find the domain of $f(x) = \dfrac{1}{\sqrt{5x-2}} - \dfrac{1}{\sqrt{9-2x}}$.

(6) (a) Show that the function $f(x) = \sin^2 x + \cos x$ is even. (b) Show that the function $g(x) = \sin x + \cos x$ is neither odd nor even. Hint for (b): Consider $x = -\pi/2$, $x = 0$ and $x = \pi/2$.

(7) Find $\lim\limits_{x\to 5} \dfrac{x-5}{(x+2)^2 - 49}$. Do not use a calculator.

(8) Find $\lim\limits_{x\to 4} \dfrac{4-x}{3-\sqrt{x+5}}$. Do not use a calculator.

(9) Find $\lim\limits_{x\to 0} x^2 \cos(1/x^2)$ using the Squeeze Rule.

(10) Find $\lim\limits_{x\to 0} \dfrac{x}{\tan(3x)}$. Do not use a calculator.

(11) Find $\lim\limits_{x\to 3^+} \dfrac{9-x^2}{|x-3|}$ and $\lim\limits_{x\to 3^-} \dfrac{9-x^2}{|x-3|}$. Do not use a calculator.

(12) Find $\lim\limits_{x\to 0} \dfrac{x^2}{1-\cos x}$ using the identity $(1-\cos x)(1+\cos x) = \sin^2 x$.

(13) Suppose f is a continuous function with domain $(-\infty, \infty)$ and f has the properties $f(2) = 5$, $f(3) = 2$, $f(4) = 7$. Show that the equation $f(x) = 4$ must have at least two different solutions.

(14) Let A and B be constants. Let $f(x)$ be given by

$$f(x) = \begin{cases} Ax^2 + 8 & \text{if } x < 2, \\ Bx - 10 & \text{if } 2 \leq x \leq 3, \\ A - x & \text{if } 3 < x. \end{cases}$$

Find the values of A and B that make f a continuous function on $(-\infty, \infty)$.

(15) Explain why the equation $x^3 + 4x^2 - 7x + 3 = 0$ must have a solution in the interval $(-10, 10)$

(16) Assume a, b, c are numbers such that $\ln a = 2$, $\ln b = 5$, $\ln c = -3$. Solve for x in the equation $\ln\left(\dfrac{a^x b^x}{c^x}\right) = 8$.

(17) Assume that a bank deposit is earning 4% annual interest compounded continuously. How long does it take for the money to triple?

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(18) A bacterial culture is increasing exponentially. At 1 pm it weighed 5 grams. At 3 pm it weighed 7 grams. What was the weight of the culture at 2 pm?

(19) Suppose $f(x) = \dfrac{1}{x^2}$. Find $f'(x)$ by computing the limit of the difference quotient.

(20) Find an equation for the line which is tangent to the curve $y = 1 + \sqrt{x}$ at the point $(4, 3)$.

(21) Find an equation for the line which is normal to the curve $y = 2 + \dfrac{1}{x}$ at the point $(1/3, 5)$.

(22) Find $\dfrac{d}{dx}\left[\dfrac{\ln x}{x^4}\right]$, $\dfrac{d}{dx}\left[(5+\sqrt{x})(\cos x)\right]$, $\dfrac{d}{dx}\left[\dfrac{(\tan x)e^x}{1+x^2}\right]$.

(23) Find the second derivative of each of the following functions of x:

$$f(x) = \sec x \ , \quad g(x) = (1+x^2)(\sin x) \ , \quad h(x) = \frac{e^x}{\sqrt{x}}$$

(24) What is the relative rate of change of $y = (3x+2)(2x+5)(4x-7)$ at $x = 1$?

(25) A golfer hits a golf ball and it flies right into the cup. The ball leaves the ground at time t_1 and it hits the cup at time t_2. The height of the ball is given by $-16t^2 + 50t - 25$ (measured in feet) at time t, where $t_1 \le t \le t_2$ and t is measured in seconds. How many seconds was the ball in the air? What was the greatest height that it reached?

Abbreviated answers to some of these problems

NOTE: These answers are not complete solutions to the problems. They are only meant to provide you with a way to check your own solutions. On an exam you must give complete solutions that show all work.

(1) (a) $x = 15/2$, (b) $x \ge 15/2$.
(2) $(x+1)^2 + (y-3)^2 = 5$.
(3) Center $= (1, -2)$, radius $= 2$.
(4) $y - 2 = (5/3)(x-1)$.
(5) The open interval $(2/5, 9/2)$.
(7) $1/14$.
(8) 6.
(9) 0.
(10) $1/3$.
(11) The limit from the right is -6. The limit from the left is 6.
(12) 2.
(14) $A = -4$, $B = 1$.
(16) $4/5$.
(17) $25(\ln 3)$ years.
(18) $\sqrt{35}$ grams.
(20) $y - 3 = (1/4)(x-4)$.
(21) $y - 5 = (1/9)(x - 1/3)$.
(24) $(3/5) + (2/7) - (4/3)$.
(25) The ball is in the air for $15/8$ seconds. Its greatest height is $225/16$ feet.