Review problems for Exam 2 in Math 135 - Spring 2004

NOTE: These are only practice problems! Your exam may have types of problems that are not represented on this sheet. It is your responsibility to study **all** of the material and to master **all** of the homework problems. The answers to the questions must be exact answers. For example, the number 1.732050808 is not a correct answer when the right answer is actually $\sqrt{3}$.

(1) Find $f'(x)$, $g'(x)$, $h'(x)$ when

$$f(x) = \sec\big(\ln(x^2+1)\big), \quad g(x) = \cos\big((4+5x)^{10}\big), \quad h(x) = e^{\sqrt{x^2+7}} .$$

(2) Find $f''(x)$ when $f(x) = \ln(x^6 + x + 2)$.

(3) Find an equation for the line tangent to the curve $xy^3 + x^4y^2 = 198$ at the point $(2, 3)$.

(4) Find the slope of the line tangent to the curve $\tan(x + y^2) = \left(\frac{6\sqrt{3}}{\pi}\right) x$ at the point $(\pi/6, \sqrt{\pi/6})$.

(5) A container has the shape of an inverted cone with a height of 15 ft and a diameter of 10 ft at the top. Water is flowing into the container at a rate of 3 cubic feet per second. How fast is the water level rising when the water is 4 ft deep?

(6) A 5 ft tall person is walking away from a streetlight that is perched 9 feet above the ground. The speed of the person is 3 feet per second. At what rate is the person's shadow lengthening?

(7) An inflated spherical balloon receives just enough extra air in order to increase its volume by 1%. By what percentage does its surface area increase when this extra air is added?

(8) Approximate $(4.001)^{-1/2}$ and $(3.999)^{-1/2}$ using differentials.

(9) Find the linearization $L(x)$ of $f(x) = \sin(x^2)$ at $\sqrt{\pi}/2$.

(10) Assume $f(x) = \big|3 - |2 - x|\big|$. Find the absolute maximum and the absolute minimum of f on the interval $[-2, 6]$. Find the absolute maximum and the absolute minimum of f on the interval $[0, 4]$.

(11) Find all relative maxima and all relative minima of $f(x) = \sin x + (\sqrt{3}) \cos x$.

(12) Assume $f(x) = x^3 + x^2$. Find a number c in the interval $(1, 3)$ such that $f'(c) = (f(3) - f(1))/(3 - 1)$.

(13) Assume $f(x) = (4x^2 + 8x + 1)e^{-x}$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the inflection points of f, the intervals where f is concave up and the intervals where f is concave down. Find the relative minima and the relative maxima of f.

(14) Assume $f(x) = (4 + 5x^2)^{-1}$. Find the inflection points of f, the intervals where f is concave up and the intervals where f is concave down.

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(15) Evaluate $\lim_{x\to-\infty} \dfrac{x^3}{\sqrt{1+7x^6}}$.

(16) Assume $f(x) = x(4-x^2)^{-1}$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the intervals where f is concave up and the intervals where f is concave down. Find the horizontal asymptotes and the vertical asymptotes.

(17) Evaluate $\lim_{x\to 0} \dfrac{\cos(3x)-1}{\cos(2x)-1}$.

(18) Evaluate $\lim_{x\to 0^+} \sqrt{x}\ln x$.

(19) A publishing house has the following rules for the printing of its books: Each page must have 1.25 inch margins on the left and the right. Each page must also have 1.75 inch margins on the top and the bottom. Finally, the printed portion of each page must be a rectangle with an area of 50 square inches. What are the dimensions of the rectangular page of least area that satisfies all of these conditions? Don't forget to check that your answer **minimizes** the area.

(20) Find the largest possible value of xy^2 when x and y are required to have the properties $x \geq 0$, $y \geq 0$, $5x + 3y = 10$.

Short answers to some of the problems

(3) $y - 3 = (-21/10)(x-2)$.
(4) $(3\sqrt{3}/(2\pi) - 1)/(2\sqrt{\pi/6})$.
(5) $27/(16\pi)$ feet per second.
(6) $15/4$ feet per second.
(7) $(2/3)\%$.
(8) $1/2 + (-1/16)(.001)$ and $1/2 + (1/16)(.001)$.
(9) $\sqrt{2}/2 + (\sqrt{2\pi}/2)(x - \sqrt{\pi}/2)$.
(10) On $[-2,6]$: max at $x = 2$, min at $x = -1, 5$. On $[0,4]$: max at $x = 2$, min at $x = 0, 4$.
(11) Relative maxima at $\pi/6 + 2n\pi$, relative minima at $7\pi/6 + 2n\pi$, where n is an integer.
(12) $(-1 + 2\sqrt{13})/3$.
(13) Increasing on $(-\sqrt{7}/2, \sqrt{7}/2)$. Decreasing on $(-\infty, -\sqrt{7}/2)$ and $(\sqrt{7}/2, \infty)$. Concave up on $(-\infty, 1 - \sqrt{11}/2)$ and $(1 + \sqrt{11}/2, \infty)$. Concave down on $(1 - \sqrt{11}/2, 1 + \sqrt{11}/2)$. Inflection points at $x = 1 - \sqrt{11}/2$ and $x = 1 + \sqrt{11}/2$. Relative minimum at $-\sqrt{7}/2$. Relative maximum at $x = \sqrt{7}/2$.
(14) Inflection points at $x = -2/\sqrt{15}$ and $x = 2/\sqrt{15}$. Concave up on $(-\infty, -2/\sqrt{15})$ and $(2/\sqrt{15}, \infty)$. Concave down on $(-2/\sqrt{15}, 2/\sqrt{15})$.
(15) $-1/\sqrt{7}$.
(16) Increasing on $(-\infty, -2)$, $(-2, 2)$ and $(2, \infty)$. Concave up on $(-\infty, -2)$ and $(0, 2)$. Concave down on $(-2, 0)$ and $(2, \infty)$. Horizontal asymptote: $y = 0$. Vertical asymptotes: $x = -2$ and $x = 2$.
(17) $9/4$.
(18) 0.
(19) $2.5 + \sqrt{5/7}\sqrt{50}$ inches by $3.5 + \sqrt{7/5}\sqrt{50}$ inches.
(20) $x = 2/3$ and $y = 20/9$.