Review sheet for the material after section 4.6

This review sheet covers only the sections that did not appear on the review sheets for the first and second exams. The final exam covers the entire semester. You should study all three review sheets. Of course, you should be able to do all of the suggested homework problems for the entire course.

1. If p dollars are charged for each pound of chocolate, then $x = 500 - 5p$ pounds will be sold. It costs $C(x) = 10 + 20x - x^2/50$ dollars to produce these x pounds of chocolate. What is the value of x that maximizes profit? Answer: 2000/9.

2. Now assume that it costs $C(x) = 10 + 20x + x^2/50$ dollars to produce x pounds of chocolate. This is not the same as the $C(x)$ in the previous problem. Find the value of x that minimizes the average cost. Answer: $10\sqrt{5}$.

3. At time t (in years) there are $20/(1+7e^{-t/20})$ million individuals in a certain population. (a) What is the size of the population when the population is growing at the maximum rate? Answer: 10 million. (b) What is the limit of the population as $t \to \infty$? Answer: 20 million. (b) What is the limit of the population as $t \to -\infty$? Answer: 0 million.

4. Find the function $f(x)$ such that $f'(x) = x^2 e^{(x^3)}$ and $f(2) = 5$.

5. Find the function $f(x)$ such that $f''(x) = e^{9x}$, $f(1) = 3$, $f'(1) = 4$.

6. Find the function $f(x)$ such that $f'(x) = \frac{d}{dx}\left[\sqrt{1 + x^3}\right]$ and $f(1) = 3$.

7. Find the following integrals: $\displaystyle\int \frac{x^3 + 5x}{2x^4 + 20x^2 + 12}\,dx$, $\displaystyle\int (\sqrt{x} + 3)(\sqrt{x} + 4)\,dx$,

$$\int (1 + e^{2x})(1 + e^{3x})\,dx, \qquad \int \frac{1 - x^2}{\sqrt{x}}\,dx, \qquad \int \sin x \cos^2 x\,dx,$$

$$\int \tan^3 x \sec^2 x\,dx, \qquad \int \tan x \sec^8 x\,dx, \qquad \int x^2 \sec^2(x^3)\,dx.$$

8. Find $\int_1^2 \sqrt{2x + 1}\,dx$. Answer: $(5^{3/2} - 3^{3/2})/3$.

9. Use $\Delta x = (b - a)/n$ and $n = 4$ to find the approximation $\sum_{k=1}^{n} f(a + k\Delta x)\Delta x$ for the area under the graph of $f(x) = (x + 1)^2$ over the interval $[a, b] = [0, 1]$. Answer: 87/32.

10. Let R be the region in the xy-plane that is bounded by the x-axis and $y = 9 - x^2$.
(a) Sketch the region R.
(b) Approximate the area of R using a Riemann sum with $x_0 = -3$, $x_1 = -1$, $x_2 = 0$, $x_3 = 3$, choosing the x_k^* to be the midpoints of the subintervals. Answer: 39.
(c) Find the exact value of the area of R. Answer: 36.

11. Assume that $s(t)$ is the position of a vehicle at time t, where t is measured in seconds and $s(t)$ is measured in feet. Assume that the acceleration $a(t)$ of the car is given by $a(t) = t + t^2$ for $0 \le t \le 100$. Suppose that at time $t = 0$ the position is 8 feet and the velocity is 5 feet/sec. Find $s(t)$ for $0 \le t \le 100$.

12. Assume that a continuous function $f(x)$ on $[1, 5]$ has the properties $\int_1^5 f(x)\,dx = 8$ and $\int_3^5 f(x)\,dx = 1$. Find $\int_1^3 f(x)\,dx$. Answer: 7.

13. Now assume that a continuous function $f(x)$ defined on $[0, 7]$ has the properties $\int_0^6 f(x)\,dx = 14$, $\int_1^7 f(x)\,dx = 19$ and $\int_1^6 f(x)\,dx = 4$. (a) Find $\int_0^1 f(x)\,dx$. Answer: 10. (b) Find $\int_6^7 f(x)\,dx$. Answer: 15. (c) Find $\int_0^7 f(x)\,dx$. Answer: 29.

14. Find $\int_0^6 |2 - x|\,dx$. Answer: 10.