TV REVIEW QUESTIONS

(1) Solve the inequality $\dfrac{x^3 - 4x^2 - x}{3 + x^2} > 0$.

(2) Let $f(x) = \tan\left(\sqrt{x^4 + 10}\right)$. Find functions f_1, f_2, f_3 such that $f = f_1 \circ f_2 \circ f_3$. Now find a different answer to this same question.

(3) Suppose that $f(x)$ is a function defined for $x \neq 0$. Assume that f is an odd function and that $\lim_{x \to 0} f(x)$ exists. Explain why we must have $\lim_{x \to 0} f(x) = 0$.

(4) Evaluate $\lim_{x \to 0} \dfrac{3x - \sin(3x)}{5x - \sin(5x)}$.

(5) Explain why the equation $2x^5 + 5x + 3 = 0$ must have a solution. Do not try to find x.

(6) A population triples in size every 5 years. How long does it take for the population to increase from 7 billion to 8 billion individuals? Assume that the growth is exponential.

(7) Show $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = -\dfrac{2}{x^3}$ using the definition of the derivative as a limit.

(8) Find an equation for the line tangent to the curve $y = \dfrac{3x + 4}{(5x + 2)(7x + 2)}$ at the point $(0, 1)$.

(9) Differentiate $f(x) = \ln\left(\sec^2 x + \sec(x^2)\right)$.

(10) Find the slope of the tangent line to the graph of $\tan y = xy^2 + 1 - \dfrac{\pi^2}{16}$ at the point $(1, \pi/4)$.

(11) Sand is pouring inside a large hour-glass at a rate of 2 cubic inches per minute. The sand pile at the bottom has the shape of a cone with $h = (7/8)r$, where h is the height and r is the radius of the base. How fast is the height increasing when the radius is 3 inches?

(12) Approximate $\sqrt{15.999}$ using differentials.

(13) Find the absolute maximum and the absolute minimum of $f(x) = \sin x + \cos x$ on the interval $[0, 2\pi]$.

(14) Let $f(x) = x^5 - x^3$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the relative extrema. Find the intervals where f is concave up and the intervals where f is concave down. Find the inflection points.

MORE QUESTIONS ON THE NEXT PAGE

(15) Find the horizontal asymptotes and the vertical asymptotes of $f(x) = \dfrac{\sqrt{1+x^6}}{x^3}$.

(16) Find the points on the parabola $y = 5x^2$ which are closest to the point $(0, 1)$ in the xy-plane.

(17) It costs $x^4 - 2x^2 + 40x + 20$ dollars to manufacture x tons of a certain product. What is the value of x that minimizes the marginal cost?

(18) A function $F(x)$ is defined on $[1, \infty)$. It has the properties $F(1) = 2$, $F'(x) = \dfrac{x+1}{\sqrt{x}}$ for $x \geq 1$. Find $F(x)$.

(19) A continuous function $f(x)$ defined on $[0, 5]$ has the properties $\int_0^5 f(x)\,dx = 12$, $\int_0^3 f(x)\,dx = 7$, $\int_2^5 f(x)\,dx = 15$. Find $\int_2^3 f(x)\,dx$.

(20) Evaluate $\int_0^{\pi/2} (\cos x)5^{\sin x}\,dx$.

ABBREVIATED ANSWERS TO SOME QUESTIONS

(1) $(2-\sqrt{5}, 0) \cup (2+\sqrt{5}, \infty)$.
(4) $27/125$.
(6) $5(\ln 8 - \ln 7)/(\ln 3)$ years.
(8) $y = 1 - (21/4)x$.
(10) $(\pi^2/16)/(2 - \pi/2)$.
(11) $2/(9\pi)$ inches per minute.
(12) $4 - (.001)/8$.
(13) Absolute maximum at $x = \pi/4$, absolute minimum at $x = 5\pi/4$.
(14) Increasing on $(-\infty, -\sqrt{3/5})$ and $(\sqrt{3/5}, \infty)$. Decreasing on $(-\sqrt{3/5}, \sqrt{3/5})$. Relative maximum at $x = -\sqrt{3/5}$. Relative minimum at $x = \sqrt{3/5}$. Concave up on $(-\sqrt{3/10}, 0)$ and $(\sqrt{3/10}, \infty)$. Concave down on $(-\infty, -\sqrt{3/10})$ and $(0, \sqrt{3/10})$. Inflection points at $x = -\sqrt{3/10}$, $x = 0$ and $x = \sqrt{3/10}$.
(15) Horizontal asymptotes: $y = 1$ and $y = -1$. Vertical asymptote: $x = 0$.
(16) $(-3/\sqrt{50}, 9/10)$ and $(3/\sqrt{50}, 9/10)$.
(17) $1/\sqrt{3}$.
(18) $(2/3)x^{3/2} + 2x^{1/2} - 2/3$.
(19) 10.
(20) $4/(\ln 5)$.