Formula sheets for the final exam in Math 135

Preliminaries: The line through the point (h, k) with slope m is given by the equation $y - k = m(x - h)$. If $a \neq 0$ and $ax^2 + bx + c = 0$ then $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. If a right triangle has sides of length a, b, c, where the side of length c is opposite the right angle, then the **Pythagorean theorem** says $a^2 + b^2 = c^2$.

Basic trig identities: $\sin(0) = 0$, $\cos(0) = 1$, $\sin^2 x + \cos^2 x = 1$, $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$, $\sin(x + 2\pi) = \sin x$, $\cos(x + 2\pi) = \cos x$, $\tan x = (\sin x)/(\cos x)$, $\cot x = (\cos x)/(\sin x)$, $\sec x = 1/(\cos x)$, $\csc x = 1/(\sin x)$.

Limits: $\lim_{x\to a} f(x) = L$ is equivalent to $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$.

Continuity: f is **continuous at** a when all of the following conditions are satisfied:
1. $f(a)$ is defined, 2. $\lim_{x\to a} f(x)$ exists, 3. $\lim_{x\to a} f(x) = f(a)$.

Derivatives: f is differentiable at x when $f'(x) = \lim_{h\to 0} \big(f(x + h) - f(x)\big)/h$ exists. The number $f'(a)$ is the slope of the tangent to the graph of $y = f(x)$ at $x = a$. The **sum and product rules** say $(f + g)' = f' + g'$ and $(fg)' = fg' + gf'$, respectively. The **quotient rule** says $(f/g)' = (gf' - fg')/(g^2)$. The **chain rule** says $\frac{d}{dx}[f(g(x))] = f'(g(x))\frac{d}{dx}[g(x)]$. $f'(c)/f(c)$ is the **relative rate of change** of $y = f(x)$ at $x = c$. $f'(c)$ is the **instantaneous rate of change**. If $s(t)$ is the **position function** then $s'(t)$ is the **velocity**, $s''(t)$ is the **acceleration** and $|s'(t)|$ is the **speed**.

Basic log and exp laws: $e^{\ln x} = x$, $\ln(e^x) = x$, $\ln(ab) = \ln a + \ln b$, $\ln(a/b) = \ln a - \ln b$, $\ln(a^b) = b \ln a$, $\ln 1 = 0$, $\ln e = 1$, $e^a e^b = e^{a+b}$, $e^a/e^b = e^{a-b}$, $(e^a)^b = e^{ab}$, $e^0 = 1$, $e^1 = e$.

Exponential growth is given by $P(t) = P_0 e^{kt}$. **Continuous compounding** of interest leads to $A(t) = Pe^{rt}$, where r is the interest rate and $A(t)$ is the balance at time t.

Calculus in economics: $C(x)$ is the **cost** of producing x units of a product. The **marginal cost** is $C'(x)$; the **average cost** is $C(x)/x$. If x units are available then $p(x)$ is the **price** consumers are willing to pay for each unit. $R(x) = xp(x)$ is the **revenue** from x units. $R'(x)$ is the **marginal revenue**. The **profit function** is $P(x) = R(x) - C(x)$.

Approximation: The **linearization** of $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$. If $y = f(x)$ then its **differential** is $dy = f'(x)dx$. This differential is approximately equal to $\Delta y = f(x + \Delta x) - f(x)$ when $dx = \Delta x$. For any variable S, ΔS is the **error**, $(\Delta S)/S$ is the **relative error** and $100\big((\Delta S)/S)\big)\%$ is the **percentage error**.

Graphing and optimization: If $f'(x) > 0$ on an interval then f is **increasing** on that interval. If $f'(x) < 0$ on an interval then f is **decreasing** on that interval. If $f''(x) > 0$ on an interval then f is **concave up** on that interval. If $f''(x) < 0$ on an interval then f is **concave down** on that interval. **Relative max and min** of a function f can occur only at **critical numbers** (numbers x in the domain of f where $f'(x) = 0$ or $f'(x)$ does not exist). **Absolute max and min** occur only at critical points or endpoints. **Inflection points** are points where the concavity of f changes sign. The line $x = a$ is a **vertical asymptote** of $y = f(x)$ if either $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$. The line $y = b$ is a **horizontal asymptote** of $y = f(x)$ if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

L'Hôpital's Rule: If $\lim_{x\to c} \frac{f(x)}{g(x)}$ is a form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$ when the second limit exists.

Integrals: If $F'(x) = f(x)$ then F is an **antiderivative** of f. $\int f(x)\,dx$ is the set of all antiderivatives of f. $F'(x) = f(x)$ is equivalent to $\int f(x)\,dx = F(x) + C$. If f is continuous and $f(x) \geq 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is the **area** under the curve $y = f(x)$ on $[a, b]$. If $a < c < b$ then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$. The **first fundamental theorem of calculus** says $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f.

Riemann sums: Assume that $a < b$ and the interval $[a, b]$ is partitioned into n subintervals $[x_{k-1}, x_k]$ of width $\Delta x_k = x_k - x_{k-1}$, where $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$. Assume also that x_k^* is chosen from $[x_{k-1}, x_k]$ as the kth interval representative. Then $\sum_{k=1}^n f(x_k^*)\Delta x_k$ is the Riemann sum associated with the function $f(x)$ and with this partition and this choice of representatives.

Differentiation rules	**Integration rules**
$\frac{d}{dx}(kx) = k$ for constant k	$\int k\,dx = kx + C$ for constant k
$\frac{d}{dx}[kf(x)] = kf'(x)$ for constant k	$\int kf(x)\,dx = k\int f(x)\,dx$ for constant k
$\frac{d}{dx}(x^k) = kx^{k-1}$ for constant k	$\int x^k\,dx = \frac{x^{k+1}}{k+1} + C$ for constant $k \neq -1$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$

Integration by substitution: If $u = u(x)$ then $\int f(u(x))u'(x)\,dx = \int f(u)\,du$ and $\int_a^b f(u(x))u'(x)\,dx = \int_{u(a)}^{u(b)} f(u)\,du\,.$