Formula Sheet for Exam 1

Preliminaries. If $r > 0$ then $(x-a)^2+(y-b)^2 = r^2$ is an equation of a circle with center (a, b) and radius r. If $x \geq 0$ then $|x| = x$. If $x < 0$ then $|x| = -x$. The inequality $|x| \leq a$ is equivalent to $-a \leq x \leq a$. Similarly, the inequality $|x| < a$ is equivalent to $-a < x < a$. A short table of exact trig values is given below:

$$\cos(0) = 1,\ \cos(\pi/6) = \sqrt{3}/2,\ \cos(\pi/4) = \sqrt{2}/2,\ \cos(\pi/3) = 1/2,\ \cos(\pi/2) = 0,$$
$$\sin(0) = 0,\ \sin(\pi/6) = 1/2,\ \sin(\pi/4) = \sqrt{2}/2,\ \sin(\pi/3) = \sqrt{3}/2,\ \sin(\pi/2) = 1.$$

This short table can be expanded through the use of the basic trig identities

$$\tan x = (\sin x)/(\cos x),\ \cot x = (\cos x)/(\sin x),\ \sec x = 1/(\cos x),\ \csc x = 1/(\sin x),$$
$$\cos(-x) = \cos(x),\ \cos(\pi - x) = -\cos(x),\ \cos(x + 2\pi) = \cos(x),$$
$$\sin(-x) = -\sin(x),\ \sin(\pi - x) = \sin(x),\ \sin(x + 2\pi) = \sin(x).$$

For example, $\cos(5\pi/6) = \cos(\pi - \pi/6) = -\cos(\pi/6) = -\sqrt{3}/2$. The identities $\cos^2 x + \sin^2 x = 1$ and $1 + \tan^2 x = \sec^2 x$ are also useful.

The line through (x_1, y_1) and (x_2, y_2) has slope $m = (y_2 - y_1)/(x_2 - x_1)$ when $x_1 \neq x_2$. The line through (h, k) with slope m is given by the equation $y - k = m(x - h)$. If two perpendicular lines have slopes m_1 and m_2, then we must have $m_2 = -1/m_1$. The line $y = mx + b$ has slope m and y-intercept b.

If $a \neq 0$ and $ax^2 + bx + c = 0$ then $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. We often use the factorizations $x^2 - a^2 = (x - a)(x + a)$ and $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$.

Limits. $\lim_{x\to c^-} f(x)$ is the limit of $f(x)$ as x approaches c from the *left* (which is the case $x < c$). $\lim_{x\to c^+} f(x)$ is the limit of $f(x)$ as x approaches c from the *right* (which is the case $x > c$). If $\lim_{x\to c^-} f(x) = L$ and $\lim_{x\to c^+} f(x) = L$ (same L), then $\lim_{x\to c} f(x) = L$. If $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$ then $\lim_{x\to c} f(x)$ does not exist. One can show $\lim_{x\to 0} \big((\sin x)/x\big) = 1$ and $\lim_{x\to 0} \big((\cos x - 1)/x\big) = 0$.

Continuity. The function f is continuous at c when all three of the following conditions are fulfilled: $f(c)$ is defined, $\lim_{x\to c} f(x)$ exists, $\lim_{x\to c} f(x) = f(c)$.

Log and exp. In addition to $b^{\log_b x} = x$, $\log_b(b^x) = x$ and $\ln x = \log_e x$ we have

$$b^x b^y = b^{x+y},\ (b^x)/(b^y) = b^{x-y},\ (b^x)^y = b^{xy},\ (ab)^x = a^x b^x,\ (a/b)^x = (a^x)/(b^x),\ b^0 = 1,\ b^1 = b,$$
$$\log_b(xy) = \log_b x + \log_b y,\ \log_b(x/y) = \log_b x - \log_b y,\ \log_b(x^p) = p \log_b x,\ \log_b(1) = 0,\ \log_b(b) = 1.$$

Exponential growth is given by the formula $P(t) = P_0 e^{kt}$. Continuous compounding of interest leads to the formula $A(t) = Pe^{rt}$, where r is the interest rate.

Derivatives. If $y = f(x)$ then $\frac{dy}{dx} = \frac{d}{dx}(f(x)) = f'(x) = \lim_{\Delta x\to 0}(f(x+\Delta x) - f(x))/(\Delta x)$ if the limit exists. The number $f'(c)$ is the slope of the tangent to the graph of $y = f(x)$ at $x = c$.

If c and n are constants then $\frac{d}{dx}(c) = 0$, $\frac{d}{dx}(x) = 1$, $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$ and $\frac{d}{dx}(x^n) = nx^{n-1}$. The sum and product rules say $(f+g)' = f' + g'$ and $(fg)' = fg' + gf'$, respectively. The quotient rule says $(f/g)' = (gf' - fg')/(g^2)$. In addition:

$$\frac{d}{dx}\sin x = \cos x,\ \frac{d}{dx}\cos x = -\sin x,\ \frac{d}{dx}\tan x = \sec^2 x,\ \frac{d}{dx}\sec x = \sec x \tan x,$$
$$\frac{d}{dx}\cot x = -\csc^2 x,\ \frac{d}{dx}\csc x = -\csc x \cot x,\ \frac{d}{dx}e^x = e^x,\ \frac{d}{dx}\ln x = \frac{1}{x}.$$

$f'(c)/f(c)$ is the relative rate of change of $y = f(x)$ at $x = c$. $f'(c)$ is the instantaneous rate of change. If $s(t)$ is the position function then $s'(t)$ is the velocity, $s''(t)$ is the acceleration and $|s'(t)|$ is the speed.