Formula Sheet for Exam 2

Preliminaries. The line through the point (h, k) with slope m is given by the equation $y - k = m(x - h)$. If $a \neq 0$ and $ax^2 + bx + c = 0$ then $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.

Log and exp. $e^{\ln x} = x$, $\ln(e^x) = x$, $\ln(a^b) = b(\ln a)$, $\log_b x = (\ln x)/(\ln b)$, $a^x = e^{x(\ln a)}$.

Derivatives. The number $f'(a)$ is the slope of the tangent to the graph of $y = f(x)$ at $x = a$. If c and n are constants then $\frac{d}{dx}(c) = 0$, $\frac{d}{dx}(x) = 1$, $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$ and $\frac{d}{dx}(x^n) = nx^{n-1}$. The **sum and product rules** say $(f+g)' = f'+g'$ and $(fg)' = fg'+gf'$, respectively. The **quotient rule** says $(f/g)' = (gf' - fg')/(g^2)$. The **chain rule** says $\dfrac{d}{dx}[f(g(x)] = f'(g(x))\dfrac{d}{dx}[g(x)]$.

When a and b are constants, we have $\dfrac{d}{dx}[a^x] = (\ln a)a^x$, $\dfrac{d}{dx}[\log_b x] = [(\ln b)x]^{-1}$. Also,

$$\frac{d}{dx}[\sin x] = \cos x,\ \frac{d}{dx}[\cos x] = -\sin x,\ \frac{d}{dx}[\tan x] = \sec^2 x,\ \frac{d}{dx}[\sec x] = \sec x \tan x,$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x,\ \frac{d}{dx}[\csc x] = -\csc x \cot x,\ \frac{d}{dx}[e^x] = e^x,\ \frac{d}{dx}[\ln x] = \frac{1}{x}.$$

The **linearization** of $f(x)$ at a is $L(x) = f(a) + f'(a)(x - a)$. If $y = f(x)$ then its **differential** is $dy = f'(x)dx$. This differential is approximately equal to $\Delta y = f(x + \Delta x) - f(x)$ when $dx = \Delta x$. For any variable S, ΔS is the **error**, $\dfrac{\Delta S}{S}$ is the **relative error** and $100\left(\dfrac{\Delta S}{S}\right)\%$ is the **percentage error**.

The line $x = a$ is a **vertical asymptote** of $y = f(x)$ if $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$. For example: If $P(x)$, $Q(x)$ are polynomials, $f(x) = P(x)/Q(x)$, $Q(a) = 0$ but $P(a) \neq 0$ then $x = a$ is a vertical asymptote of $y = f(x)$. The line $y = b$ is a **horizontal asymptote** of $y = f(x)$ if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

A function f is **increasing** on intervals where f' is positive. It is **decreasing** on intervals where f' is negative. It is **concave upward** on intervals where f'' is positive. It is **concave downward** on intervals where f'' is negative. A **critical number** of a function f is any point x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist.

To find the **absolute maxima** and the **absolute minima** of a continuous function on the interval $[a, b]$: Evaluate the function at a, b and all of its critical numbers between a and b. Determine the max and min of these evaluations.

First derivative test at a critical number c of f where f is continuous: f has a relative maximum at c if the sign of $f'(x)$ goes from $+$ to $-$ as x moves across c from left to right. f has a relative minimum at c if the sign of $f'(x)$ goes from $-$ to $+$ as x moves across c from left to right.

Second derivative test at a critical point c of f: If $f'(c) = 0$ and $f''(c) < 0$ then f has a relative maximum at c. If $f'(c) = 0$ and $f''(c) > 0$ then f has a relative minimum at c.
Suppose that $(c, f(c))$ is a point on the graph of a differentiable function f. If the concavity of $y = f(x)$ changes as x moves across c then $(c, f(c))$ is an **inflection point**.

L'Hôpital's Rule. If $\lim_{x\to c} \dfrac{f(x)}{g(x)}$ is a form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \lim_{x\to c} \dfrac{f'(x)}{g'(x)}$ when the second limit exists.