

Review Problems for Exam 1 in Math 135 - Spring 2005

NOTE: Your exam may have types of problems that are not represented below. The choice of midterm exam questions is up to the individual instructor. In order to be prepared for many types of exam questions, you should study all of the textbook material and make sure you can do all of the suggested homework problems on the course web page. Answers to exam questions must be exact, not calculator approximations. For example, the number 1.732050808 is not a correct answer when the right answer is actually $\sqrt{3}$.

(1) Find an equation for the tangent line to the graph of $y = \frac{x}{1+x^2}$ at the point $(2, 2/5)$ in the xy plane.

(2) Find an equation for the normal line to the graph of $y = \frac{1}{\sqrt{x}}$ at the point $(4, 1/2)$ in the xy plane.

(3) Find all values of x such that $f(x) = x^3 - 15x^2 + 63x + 23$ has a horizontal tangent at $(x, f(x))$.

(4) The relative rate of change of $y = f(x)g(x)h(x)$ at $x = x_0$ is given by

$$\frac{f'(x_0)}{f(x_0)} + \frac{g'(x_0)}{g(x_0)} + \frac{h'(x_0)}{h(x_0)}.$$

Explain why this formula is a consequence of the Product Rule. Use the formula to find the relative rate of change of $y = (2x + 3)(7x + 1)(9x + 2)$ at $x = 5$.

(5) A baseball batter hits the ball in such a way that it flies straight up. The catcher grabs the baseball (on its way down) when it is 5 feet above home plate. The height $h(t)$ of the ball at time t seconds is given by

$$h(t) = -16t^2 + 48t + 3 \text{ feet} \quad \text{for } t_0 \leq t \leq t_1,$$

where t_0 is the instant the bat hits the ball, and t_1 is the moment when the ball lands in the catcher's mitt. What is the acceleration of the ball when $t_0 < t < t_1$? What is the speed of the ball when it hits the catcher's glove? What is the greatest height reached by the baseball?

(6) Solve the inequality $|5x - 1| \leq 2$. Solve the inequality $|2x - 7| > 5$.

(7) Suppose that n is a natural number. Find the values of n for which $\sin(x^n)$ is an odd function. Find the values of n for which $\sin(x^n)$ is an even function. How would your answer change if \sin were replaced by \cos ?

(8) Assume $f(x) = \sqrt{2x+1}$. Find $f'(3)$ by computing the limit of the difference quotients.

(9) Assume that A, B, C, D are constants. Suppose that f is the function given by

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < 2, \\ A & \text{if } x = 2, \\ Bx + C & \text{if } 2 < x < 3, \\ D & \text{if } x = 3, \\ 5x + 2 & \text{if } x > 3. \end{cases}$$

Find A, B, C, D such that f is continuous.

(10) Explain why the equation $x^7 + 5x^2 + 10x = 2$ must have a solution.

(11) Find the domain of $f(x) = \sqrt{10 - \sqrt{2x - 3}}$.

(12) Find $\lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{5x})}{\sqrt{x}}$.

(13) Find $\lim_{x \rightarrow 4} \frac{3x^2 - 14x + 8}{2x^2 - 9x + 4}$.

(14) Find $\lim_{x \rightarrow 3} \frac{\sqrt{2x+1} - \sqrt{7}}{3-x}$.

(15) Solve $\log_{\sqrt{x}} 27 = 3$.

(16) A bacterial culture triples in size every 4 hours. There were 10^6 germs at time $t = 0$ hours. At what time t are there going to be 10^7 germs in the culture?

(17) Show $\lim_{x \rightarrow 0} x^2(\sin(1/x) + \cos(1/x^2)) = 0$ using the Squeeze Rule.

(18) Assume $f(x) = (\sec x)(1 + 6x^8)$ and $g(x) = \frac{\ln x}{e^x + x^4}$. Find $f'(x)$, $f''(x)$, and $g'(x)$.

(19) Assume $f(x) = (\sin x)(\cos x)$. Find $f'(x)$ and $f''(x)$.

(20) Find the center and the radius of the circle $2x^2 + 2y^2 - 12x + 8y + 20 = 0$.