Short answers to the review problems for Exam 1

(1) $y - 2/5 = -(3/25)(x - 2)$.

(2) $y - 1/2 = 16(x - 4)$.

(3) $x = 3$ and $x = 7$.

(4) Answer to first part: If $k = fgh$ then $\dfrac{k'}{k} = \dfrac{f'gh + fg'h + fgh'}{fgh} = \dfrac{f'}{f} + \dfrac{g'}{g} + \dfrac{h'}{h}$. Answer to second part: $\dfrac{2}{13} + \dfrac{7}{36} + \dfrac{9}{47}$.

(5) The answers are: -32 ft/sec^2, $8\sqrt{34}$ ft/sec, 39 ft, respectively.

(6) The answers are: $[-1/5, 3/5]$ and $(-\infty, 1) \cup (6, \infty)$, respectively.

(7) $\sin(x^n)$ is an odd function when n is odd. $\sin(x^n)$ is an even function when n is even. $\cos(x^n)$ is an even function for every natural number n.

(8) The difference quotient is

$$\frac{\sqrt{7 + 2\Delta x} - \sqrt{7}}{\Delta x} = \frac{\sqrt{7 + 2\Delta x} - \sqrt{7}}{\Delta x} \cdot \frac{\sqrt{7 + 2\Delta x} + \sqrt{7}}{\sqrt{7 + 2\Delta x} + \sqrt{7}}$$
$$= \frac{(7 + 2\Delta x) - 7}{\Delta x(\sqrt{7 + 2\Delta x} + \sqrt{7})} = \frac{2}{\sqrt{7 + 2\Delta x} + \sqrt{7}}.$$

Letting Δx approach 0, we obtain $f'(3) = 1/\sqrt{7}$.

(9) Continuity at 2 implies $5 = A = 2B + C$. Continuity at 3 implies $17 = D = 3B + C$. We can solve these two equations involving the unknowns B, C. The result is $B = 12$, $C = -19$.

(10) Let f be the function $f(x) = x^7 + 5x^2 + 10x$. Since $f(-10) < 2$, $f(10) > 2$ and f is continuous, the Intermediate Value Theorem tells us that the equation $f(x) = 2$ must have a solution in the interval $(-10, 10)$. The number 10 was chosen to make the arithmetic easy.

(11) $[3/2, 103/2]$.

(12) $\sqrt{5}$.

(13) $10/7$.

(14) $-1/\sqrt{7}$.

(15) $x = 9$.

(16) $t = \dfrac{4 \ln 10}{\ln 3}$.

(17) The limit is 0 because the identities

$$\lim_{x \to 0}(-2x^2) = 0 = \lim_{x \to 0}(2x^2)$$

tell us that the Squeeze Rule can be applied to the inequalities

$$-2x^2 \le x^2\big(\sin(1/x) + \cos(1/x^2)\big) \le 2x^2.$$

(18) $f'(x) = (\sec x)\big((\tan x)(1 + 6x^8) + 48x^7\big)$, $f''(x)$ equals

$$(\sec x)\bigg((\tan x)\big((\tan x)(1 + 6x^8) + 48x^7\big) + \big((\sec^2 x)(1 + 6x^8) + (\tan x)48x^7 + 336x^6\big)\bigg),$$

$$g'(x) = \frac{(1/x)(e^x + x^4) - (\ln x)(e^x + 4x^3)}{(e^x + x^4)^2}.$$

(19) $f'(x) = \cos^2 x - \sin^2 x$, $f''(x) = -4(\sin x)(\cos x)$.

(20) The center is $(3, -2)$. The radius is $\sqrt{3}$.