

Review problems for Exam 2 in Math 135 - Spring 2005

NOTE: These are only practice problems! Your exam may have types of problems that are not represented on this sheet. It is your responsibility to study **all** of the material and to master **all** of the homework problems. The answers to the questions must be exact answers. For example, the number 1.732050808 is not a correct answer when the right answer is actually $\sqrt{3}$.

- (1) Find $f'(x)$ and $g'(x)$ when $f(x) = \ln(5^x + \sin x)$ and $g(x) = \sec(\sqrt{x^2 + 1})$.
- (2) Find $f''(x)$ when $f(x) = \sqrt{2 + \cos x}$.
- (3) Find $\frac{d}{dx} [x^{\sin x}]$.
- (4) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x^2 + 3y^2 = 1$.
- (5) Find an equation for the line tangent to the curve $x^2y^3 + x^3y = 126$ at the point $(3, 2)$.
- (6) A balloon is being inflated at a rate of 3 cubic inches per second. How fast is its surface area increasing when the radius is 5 inches?
- (7) A 5 foot tall person is walking towards a streetlight that is perched 15 feet above the ground. The person's shadow is shortening at a rate of 2 feet per second. What is the speed of the person?
- (8) Find the linearization $L(x)$ of $f(x) = 5 + x^3$ at 2.
- (9) Approximate $\sqrt{80.99}$ using differentials and the fact $\sqrt{81} = 9$.
- (10) Find the critical numbers, the absolute extrema and the relative extrema of $f(x) = |4 - |x - 2||$.
- (11) Consider the function $f(x) = |x|$. Show that there is no number c such that $f'(c) = (f(3) - f(-2))/(3 - (-2))$. Explain why this does not contradict the Mean Value Theorem.
- (12) Assume $f(x) = 2x^5 - 5x^3 + 5x$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the intervals where f is concave up and the intervals where f is concave down. Find the inflection points and the relative extrema.
- (13) Assume $f(x) = x \ln x$ for $x > 0$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the absolute extrema of f . Is it possible to define $f(0)$ in such a way that f becomes right continuous at 0?
- (14) Assume $f(x) = xe^{-x}$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the intervals where f is concave up and the intervals where f is concave down. Find the inflection points and the relative extrema.
- (15) Assume $f(x) = x(x^2 - 2)^{-1}$. Find the intervals where f is increasing and the intervals where f is decreasing. Find the intervals where f is concave up and the intervals where f is concave down. Find the inflection points, the horizontal asymptotes and the vertical asymptotes of f .

(16) Evaluate $\lim_{x \rightarrow -\infty} \frac{\sqrt{2 + 5x^2}}{x}$.

(17) Find $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{1 - \cos(3x)}$.

(18) Find $\lim_{x \rightarrow 0^+} (1 + 2x)^{1/x}$.

(19) Consider the collection of all triangles ABC such that the length of AB equals 3 inches and the length of AC is also equal to 3 inches. What is the largest possible area of such a triangle?

(20) Consider the collection of all cylindrical cans with volume equal to 50 cubic inches. What is the smallest possible surface area of such a can? The surface area includes the top and the bottom of the can, as well as the lateral area.