(25) 1. Calculate the following limits. Give a brief justification of your answers without reference to calculator computations or graphing.

(a) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

(b) $\lim_{x \to 0} \frac{\sin 3x}{\tan 2x}$

(c) $\lim_{x \to \infty} \frac{5x^4 + 7x^3 + 2x^2 + 10}{7x^4 + 5x^3 + 2x + 1}$

(d) $\lim_{x \to \infty} \frac{e^{x/2}}{x^2}$

(e) $\lim_{x \to 3} \frac{x^2}{\ln x}$

(10) 2. Compute the derivative of $\sqrt{x + 3}$ **directly from the definition.**

(25) 3. Compute the derivatives with respect to x of the following functions. Algebraic simplification of the answers need not be performed.

(a) $\ln(x) \cos(2x)$

(b) $\frac{e^x}{2x^3 + x}$

(c) $\int_0^x \sec t \, dt$

(d) $\int_0^{x^3} e^{t^2} \, dt$

(e) $\sqrt{x^4 + 3}$

(10) 4. Suppose that f is a function with first and second derivatives. Suppose in addition that the following values are known: $f'(0) = 2$, $f'(1) = 3$, $f''(0) = 4$, and $f''(1) = 5$. If $g(x) = f(\ln(x))$, what are $g'(1)$ and $g''(1)$?

(15) 5. Find the following indefinite integrals:

(a) $\int (x^3 + \frac{3}{x} + \cos x) \, dx$

(b) $\int (2x + 1) \sec^2(x^2 + x) \, dx$

(c) $\int \frac{6x^2 - 4}{(x^3 - 2x + 1)^3} \, dx$

(18) 6. Compute the following:

(a) $\displaystyle\int_1^2 \frac{\sqrt{x}+8}{x}\,dx$

(b) The area under the graph of $y = 2 + x^2 + \sin x$ on the interval $[0, \pi]$.

(c) $\displaystyle\int_0^{\pi} x^2 \sin(x^3)\,dx$

(10) 7. In the following, A and B are constants. Let f be the function defined by

$$f(x) = \begin{cases} x^3 + Ax & \text{if } x \le 1 \\ Bx^2 + 2 & \text{if } x > 1 \end{cases}.$$

(a) What is $\lim_{x \to 1^-} f(x)$?

(b) What is $\lim_{x \to 1^+} f(x)$?

(c) How must A and B be related if $f(x)$ is continuous at $x = 1$?

(d) What must the values of A and B be if $f(x)$ is differentiable at $x = 1$?

(9) 8. Use the linearization of $\tan x$ at $x = \pi/4$ to estimate the value of $\tan(\pi/4 + 0.13)$.

(10) 9. Find an equation for the tangent line to the graph of $2x^3y^2 + x^2y^3 = 16$ at the point $(1, 2)$.

(10) 10. In this problem, assume that coordinates are given in feet. A point is moving along the x-axis in such a way that its acceleration at time t is $t + \cos 2t$ ft/sec^2.

(a) Suppose the velocity of the point at $t = 0$ is 3 ft/sec. Describe the velocity of the point as a function of t.

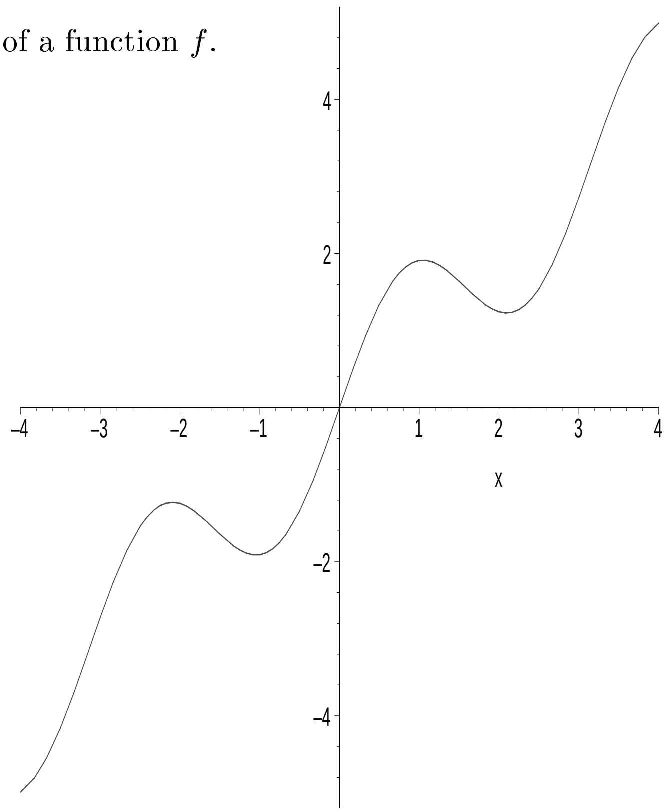
(b) Suppose the coordinate of the point at $t = 0$ is 10. Describe the position of the point at time t.

(8) 11. Compute the value of the Riemann sum for the function 2^x on the interval $[-1, 2]$ using the partition -1, 0, 1, 2 and taking as the representative points the right endpoint of each subinterval.

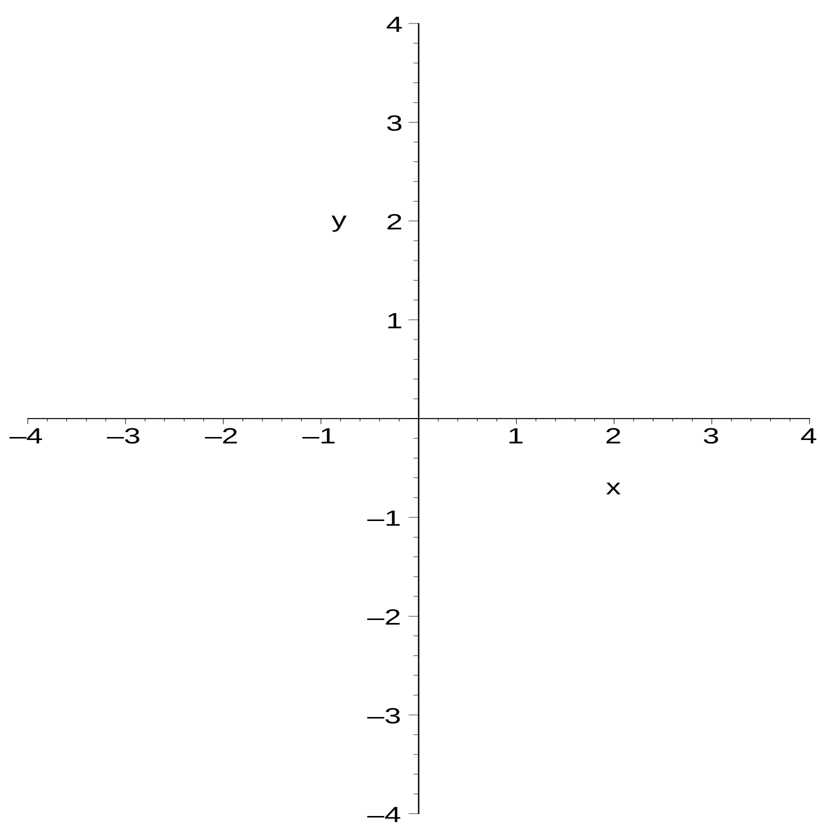
(10) 12. What point on the graph of $y = \sqrt{x}$ is closest to the point $(3, 0)$?

(10) 13. Find equations for all horizontal and vertical asymptotes of the function $\dfrac{4e^{-x} + 3}{7e^{-x} - 2}$.

(10) 14. Here is the graph of a function f.



On the axes below, sketch the graph of the derivative of f.



(10) 15. You may find it hard, but imagine you are watching a balloon in the shape of a cube being inflated. At a certain moment the volume of the balloon is 8 cubic feet and the volume is increasing at the rate of 0.3 cubic feet per minute. How fast is the surface area of the balloon increasing at that moment?

(10) 16. In the space below, sketch the graph of a function f with the following properties: $f(x)$ is defined and differentiable for all real numbers x except $x = -3$ and $x = 2$. The graph of f has vertical asymptotes at $x = -3$ and $x = 2$.

$$\lim_{x \to \infty} f(x) = -1 \quad \text{and} \quad \lim_{x \to -\infty} f(x) = 2.$$

The graph of f is concave down on the intervals $(-\infty, -3)$ and $(2, \infty)$ and the graph is concave up on the interval $(-3, 2)$.