

- (16) 1. Calculate the following limits. Give a brief justification of your answers without reference to calculator computations or graphing.

(a)  $\lim_{x \rightarrow 0} \frac{4x}{\sin 3x}$

(b)  $\lim_{x \rightarrow -\infty} \frac{3x^4 + x^3 - 2x^2 + 10}{7x^5 - 4x^3 + 2x + 1}$

(c)  $\lim_{x \rightarrow 0} \frac{\cos(2x) - 1}{x^2}$

(d)  $\lim_{x \rightarrow -3^-} \frac{|2x + 6|}{x + 3}$

- (10) 2. Compute the derivative of  $\frac{2}{x-1}$  **directly from the definition**.

- (16) 3. Compute the derivatives with respect to  $x$  of the following functions. Algebraic simplification of the answers need not be performed.

(a)  $e^x \ln(2x)$

(b)  $\frac{\sin x}{x^3 + 2x}$

(c)  $\int_x^0 \sec t \, dt$

(d)  $\int_0^{x^2+x} \sin(2t) \, dt$

- (10) 4. Suppose that  $f$  is a function with first and second derivatives. Suppose in addition that the following values are known:  $f(1) = 0$ ,  $f'(1) = 3$ , and  $f''(1) = 5$ . If  $g(x) = e^{f(x)}$ , what are  $g'(1)$  and  $g''(1)$ ?

(15) 5. Find the following indefinite integrals:

(a)  $\int \left( x^4 + \sec(x) \tan(x) - \frac{2}{x} \right) dx$

(b)  $\int 3x^2 \sin(x^3 + 1) dx$

(c)  $\int \cos(x) e^{\sin(x)} dx$

(18) 6. Compute the following:

(a)  $\int_1^2 \frac{x^3 + 5}{x} dx$

(b) The area under the graph of  $y = 4x + e^x$  on the interval  $[0, 2]$ .

(c)  $\int_0^1 \frac{x}{(x^2 + 1)^2} dx$

(18) 7. In the following,  $A$  and  $B$  are constants. Let  $f$  be the function defined by

$$f(x) = \begin{cases} x^2 + Ax + 3 & \text{if } x < -1 \\ x^3 & \text{if } -1 \leq x < 1. \\ Bx^2 + 2x & \text{if } 1 \leq x \end{cases}$$

(a) What are  $f(-1)$  and  $f(1)$ ?

(b) What are  $\lim_{x \rightarrow -1^-} f(x)$  and  $\lim_{x \rightarrow 1^-} f(x)$ ?

(c) Assume that  $f(x)$  is continuous for all  $x$ . What are  $A$  and  $B$ ?

(d) Suppose that  $A$  and  $B$  have the values determined in (c). Is  $f(x)$  differentiable for all  $x$ ?

(8) 8. Let  $f(x) = 3x + \ln(x^2 - 3)$ . Use the linearization of  $f(x)$  at  $x = 2$  to estimate the value of  $f(1.97)$ .

(12) 9. Let  $f(x) = 3x^5 - 5x^4 - 20x^3 + 13x - 3$ . At which point of inflection of the graph of  $f(x)$  does the tangent line have positive slope?

(16) 10. A particle is moving along the  $x$ -axis in such a way that its position, measured in feet, is

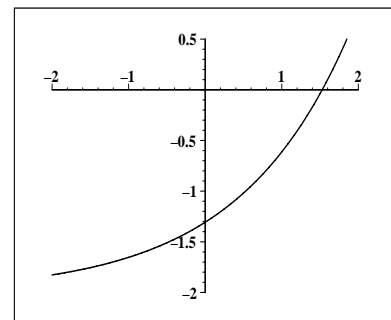
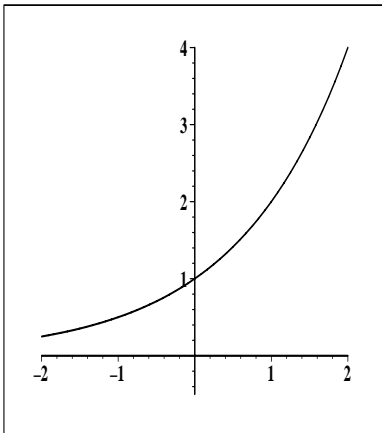
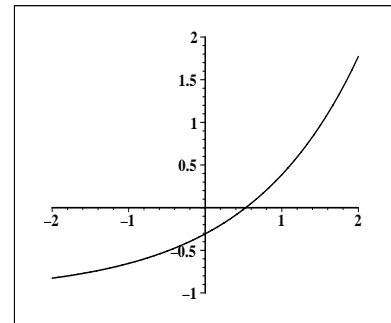
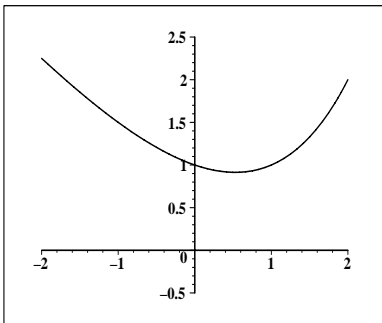
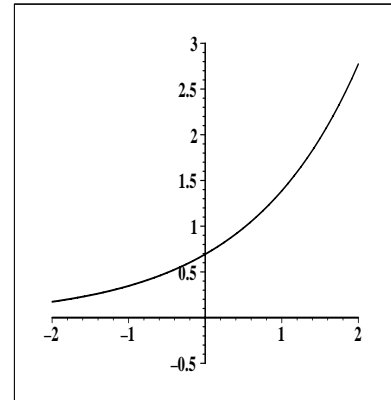
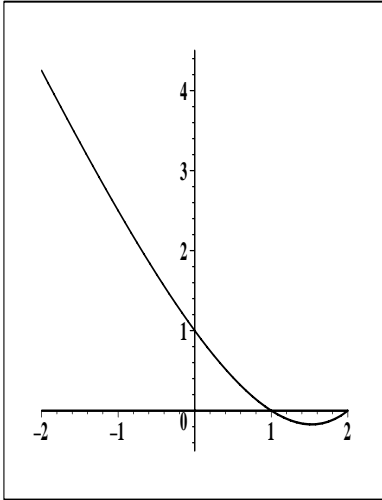
$$s(t) = 6t^4 + 12t^3 - 72t^2 + 12t - 7,$$

where  $t$  is the time in seconds. During the interval  $-1 \leq t \leq 2$ , what are the absolute maximum and the absolute minimum values of the velocity of the particle?

(8) 11. Compute the value of the Riemann sum for the function  $x^2$  on the interval  $[-2, 4]$  using the partition  $-2, 0, 2, 4$  and taking as the representative points the midpoint of each subinterval.

(15) 12. An apartment complex has 90 units. When the monthly rent for each unit is \$600, all units are occupied. Experience indicates that for each \$20-per-month increase in rent, 3 units will become vacant. Each rented apartment costs the owners of the complex \$320 per month to maintain. What monthly rent should be charged to maximize the owners' profits?

- (10) 13. On the left below are the graphs of three functions and on the right are the graphs of their derivatives in some order. Draw a line from each graph on the left to its derivative on the right.



- (18) 14. In the following, assume that distances are measured in feet and time in seconds. A particle is moving along the parabola with equation  $y = x^2$ . At the moment the particle reaches the point  $(1, 1)$ , the  $x$ -coordinate of the point is increasing at the rate of 3 feet/second.
- a) At this moment, what is the rate of change of the  $y$ -coordinate of the point?
- b) At this moment, how far is the particle from the point  $(5, -2)$ ?
- c) At this moment, how fast is the distance of the particle from the point  $(5, -2)$  changing? Is the distance increasing or decreasing?

- (10) 15. On the axes below, sketch the graph of a function  $f$  with the following properties:  $f(x)$  is defined and differentiable for all real numbers  $x$  except  $x = -1$  and  $x = 3$ . The graph of  $f$  has vertical asymptotes at  $x = -1$  and  $x = 3$ .

$$\lim_{x \rightarrow \infty} f(x) = -1 \quad \text{and} \quad \lim_{x \rightarrow -\infty} f(x) = 2.$$

The function  $f$  is increasing on the interval  $(-1, 3)$ . There is a point of inflection at  $x = 1$ . The graph is concave up on  $(-\infty, -1)$ . There is a relative maximum at  $x = 5$ .

