

Mathematics 138 Spring 2003, Review Sheet I

1. Compute these integrals:

$$(a) \int_0^\pi (\sin x + \cos x)^2 dx \quad (c) \int_0^\infty x^2 e^{-x} dx \quad (e) \int_0^\infty \frac{e^x}{1 + e^{2x}} dx$$

$$(b) \int_2^\infty \frac{1}{x \ln^2(x)} dx \quad (d) \int_0^1 e^x \sqrt{4 + e^{2x}} dx$$

2. Sketch the following regions and compute their areas.

(a) The region between the parabolas $y = x^2 - 3$ and $y = -4 + 3x - 3x^2$.

(b) The region between the curves $y = \sin x$ and $y = \sin(2x)$, from $x = 0$ to the next point of intersection of the two curves.

3. Find the average value of $(x + e^x)^2$ over the interval $[0, 1]$.

4. If we wish to estimate $\int_1^2 \frac{1}{x} dx$ numerically using the Trapezoidal Rule, how many intervals will we need to ensure an accuracy within 10^{-6} ? And how many will we need to ensure the same degree of accuracy with Simpson's Rule?

Use the standard error estimates for these rules.

5. Find the Taylor polynomial of degree 3 for $\ln x$ with center $a = 1$ and use it to estimate $\ln 2$. Use the Remainder theorem to estimate the error involved.

6. Find the Taylor polynomial of degree 6 for e^{-x^2} and use it to estimate the integral $\int_0^1 e^{-x^2} dx$. (Optional – Can you use the Remainder Theorem to estimate the error involved in this procedure?)

7. Solve the initial value problem $y' = xy + x + y + 1$ with $y(0) = 1$, and compute $y(1)$, exactly.

8. (a) Suppose you are studying a body of material, at a rate of 5% of the unlearned material per day. (Note: this refers to the instantaneous rate of learning, rather than the net daily rate). Derive the following differential equation for the amount $y(t)$ learned after t days, and solve it:

$$y' = .05(1 - y)$$

Then use this model to predict how long it will take to learn half the material, starting from scratch.

(b) Assuming the same type of learning model, if you have a week to learn a body of new material, what percentage would you have to master on a *daily* basis to reach the 95% competency level by the end of the week?

9. A 100 gallon water tank has two inflow pipes and one outflow pipe. The first inflow pipe delivers brine at a rate of 3 gallons per minute, and the brine contains 2lbs per gallon of salt; the second pipe delivers brine at the rate of 2 gallons per minute, and the brine contains 3 lbs per gallon of salt; the outflow pipe takes off 5 gallons per minute of thoroughly mixed salt water.

Write down a differential equation for the amount of salt in the tank after t minutes, and predict the amount after an hour. Predict the “long-term” effect: how salty will the water become in the long run?

Make your own “common sense” prediction of the final answer, and compare it to the computed answer.

10. The population of Nepal was growing at the rate of 2.5% per year, the last time I looked (which was several years ago). At that rate, how long would it take the population to double? (Rough answer: less than 30 years.)

11. A sample of cloth from a museum has 20% less carbon-14 in it than a comparable modern sample. How old is it?