Math 138: 10–12 — Spring 1997
MW6 HCK–138
Prof. Bumby

My other class meets MW5 on this Campus, so I will be holding office hours in the Math Department outpost on the second floor of the Chemistry building (my room is 207B) during the fourth period on those days. You may also make an appointment to meet me at other times at my regular office, Hill 438, on Busch Campus. I read electronic mail almost every day, so that is the most reliable way to contact me: the address is *bumby@math* from any machine in the *rutgers.edu* domain. There is also an answering machine on my phone, so you may leave a telephone message by phoning 445-0277.

Starting with the lecture of February 24, we will be using a supplement to the Calculus textbook. At this point, I haven't seen the supplement in the form that it is being distributed, but I have been told that it is consists of material from four sources (that I do have) bound together in a single booklet. The price of this supplement is $4,and it is available from the Mathematics Department. The primary distribution point is the Undergraduate Office, Room 303 in Hill Center on Busch Campus. That office is open from 8:30 AM to 4:30 PM Monday thru Friday. Copies should also be available at the Department Administrative Office in Chemistry 101 on the Douglass Campus. That office is open from 8:30 AM to 12:30 PM on Monday, Wednesday and Friday.

The portion of the supplement that we will need in this segment of the course is Chapter 16 of the longer version of the present textbook for topics in Differential Equations. References below will use the textbook page numbers printed at the top of the page.

A grading policy for the course has now been established. Each of the four class exams will be graded on a basis of 80 points. The final will be graded on a basis of 180 points, giving a maximum score of 500 points for the course. Since letter grades are supposed to measure qualitative distinctions, I will attempt to assure that there is a significant difference between scores giving different grades.

Since other considerations led to an early date for the first exam, some material not specifically involved with Differential Equations will also be included in this segment. These topics are taken chapters 7 and 8 of the Calculus text.

Most of the time in this segment of the course, we will be concerned with differential equations whose solutions can be determined in closed form. Several different types of equations with this property are known. Many of these types are characterized by rather general criteria. Some equations will belong to more than one type. Of course, in this case, we simply have more than one way to discover the *same* set of solutions. The second-order linear equations considered in section 16.5 will all have constant coefficients, but none of the other families will suffer this type of restriction.

This segment of the course leads to an exam on March 05. Here are the details of lecture schedule and some exercises to help you focus your preparation for the exam.

section	lecture date	page	problems
5.3	February 10	356	4, 6, 20, 30, 46
5.4		367	20, 22, 26, 28, 40, 44, 46, 56
7.6	February 12	523	2, 6, 8, 32, 38, 46
7.7	February 17	532	10, 12, 14, 24
7.8		543	8, 12, 24, 28, 32
8.7	February 19	604	6, 10, 12, 16, 28, 32, 40
16.4	February 24	1103	4, 8, 16, 18, 48, 50
16.5	February 26	1112	8, 16, 18, 30, 34
16.6	March 3	1120	6, 8, 14, 18, 20

Some of the topics covered here are not quite what they seem. In section 8.7 one meets Taylor and Maclaurin polynomials. The distinction is a minor one: if one uses $c = 0$ in the Taylor polynomial, the result is called a Maclaurin polynomial. This hardly seems to be a distinction requiring a separate name! Indeed, if one has the dexterity to correctly change variables, defining $g(u) = f(c+u)$, then, the Maclaurin series for the function $g(u)$ gives the Taylor series for the function $f(x)$ at $x = c$. It is important to note that one needs to transform the function $f(x)$ and not merely its Maclaurin series. Whatever one calls it, its important property is almost buried in the statement of Taylor's theorem (Theorem 8.19) on page 602: the form of the error term is such that no other polynomial can approximate the function this well as $x \to c$. It is this property that allow one to conclude that substituting $x = u^2$ in the Maclaurin polynomial of degree n for e^x gives the Maclaurin polynomial of degree $2n$ for e^{u^2}. You are invited to compare this with the determination of the coefficients from the formula. This error estimate is also used to show that functions like exponential and trigonometric functions and their inverses can be approximated to any required accuracy by polynomials. This is what allows your calculator to claim to be able to compute these functions. Digital computation methods are based on things that can be done exactly, such as addition and multiplication of integers. (It was proved to you that these operations can be done exactly in elementary school, when you learned methods for doing these operations with pencil and paper.) All operations that claim to be computing with real numbers are approximate, and must be designed so that the error is acceptable for all reasonable use. (Here are two examples of *unreasonable* uses of these functions on a calculator: (1) $e^{1}000$; (2) $\sin(10^{2}0)$. Example (1) is unreasonable because the answer is too large to be represented, numbers must fit into registers designed into the hardware that limilt the size of numbers to something like $10^{1}00$. Example (2) is unreasonable because numbers like $10^{2}0$ are represete in a *floating point* representation that separately keeps information about size and precison. Although $10^{2}0$ is of an acceptable size, only about the first 15 decimal places of the representation of any number are kept. For numbers of this size, this means that the same representation is used for all numbers in an interval whose length is several thousand. Since the trigonometric functions have a period of π, many periods are included, and $\sin(x)$ covers the entire interval $[-1, 1]$ many times for x in this domain.)

Section 16.6 introduces the method of *variation of parameters* to solve inhomogeneous differential equations. This is a powerful method, but someone tedious to use for hand computation. You should be aware of its existence, but exercises and exam problems will only involve equations where it is possible to guess the form of the solution and apply the method of *undetermined coefficients*.