Exercises 3.8

Last time, we did #19.

A classical example of **logarithmic differentiation** is

$$\frac{d}{dx}x^x$$

Exercise #52 is a variant on

$$e = \lim_{x \to 0}(1 + x)^{1/x}$$

which was proved by considering the derivative of $\ln x$ at $x = 1$.

Exercises 3.10

Last time, we did #25.

Other classical examples are #12 and #21.

Differentials

Writing the derivative in the form

$$\frac{dy}{dx}$$

begs for an interpretation as a fraction. If we are considering the value at a point (x_0, y_0) of the graph showing the relation between x and y, then dx and dy can be interpreted as changes in x and y on the tangent line. Indeed, when equated to the derivative, this is an equation of the tangent line. As long as the tangent is a good approximation to the curve, these differentials can be used to find changes in y caused by **small** changes in x.

Exercises 3.11

We will do #9, #22, #25, #33, #35.

Newton's method

Following the tangent line is a good way to improve approximations to the solution of $f(x) = 0$,

The textbook treatment brags about the remarkable efficiency of the method, but only asks you for four decimal places. Your calculator is fast enough that it can give fifteen place accuracy in an acceptable time using the bisection method mentioned in connection with the intermediate value theorem.

Exercise 4.9#5

Solve

$$x^3 + 2x - 4 = 0 \quad \text{starting from} \quad x_1 = 1$$

Here are the values given by Maple: 1., 1.2, 1.179746835. Then, 1.179509057 and 1.179509025. Three more steps give

$$1.17950902460291676855760349419214272490287034645 67$$

correct to 50 decimal places.

A picture of newton's method