Math 151, Fall 2014, Review Problems for the Final Exam

Your final exam is cumulative. Consequently, you should also study the review problems for the two midterm exams. The review problems that you see below deal with topics that were not well represented in the reviews for the two midterm exams. Some of these final exam review problems resemble actual questions on your final exam. Some of these final exam review problems do not resemble any questions on your final exam.

(1) A continuous function \( f(x) \) on the interval \([1, 10]\) has the properties
\[
\int_1^8 f(x) \, dx = 14, \quad \int_4^{10} f(x) \, dx = 7, \quad \int_1^{10} f(x) \, dx = 2.
\]
Find \( \int_4^8 f(x) \, dx \).

(2) Evaluate \( \frac{d}{dx} \int_0^x e^{t^2} \, dt \).

(3) Use L’Hôpital’s Rule to show \( \lim_{x \to \infty} \left( 1 + \frac{3}{x} \right)^x = e^3 \).

(4) A bacterial population quadruples in size every 7 days. How many days does it take for this population to triple in size?

(5) Evaluate \( \int \frac{x^3 \, dx}{\sqrt{1 - x^8}} \).

(6) Evaluate \( \int \frac{x^2 \, dx}{1 + x^6} \).

(7) Let \( f(x) \) be defined for \( x > 0 \) by \( f(x) = x^2 \ln x \). Find the intervals where \( f(x) \) is concave up and the intervals where \( f(x) \) is concave down.

(8) Evaluate \( \int_{-2}^{5} |x - 1| \, dx \).

(9) Find the points on the curve \( x = 2y^2 \) which are closest to the point \((10, 0)\) in the \(xy\)-plane.

(10) Find the second derivative of \( f(x) = \sin^{-1} x \).

(11) Find the area of the region bounded by \( x = y^2 \) and \( x = y + 2 \).

(12) Find \( \sum_{i=1000}^{2000} i \).