Math 152:10–12 — Fall 1999
TF3 CHM-201
Prof. Bumby

The Course Web Page. Copies of all handouts and material prepared for lectures will be made available on the Web. The *URL* of the course page is

http://www.math.rutgers.edu/~bumby/152f99/,

where the final "/" indicates that this is the index file of a directory. The two generations of parent directories also contain index files that you may find useful. My office hours appear on my general page, and the department page is a gateway to everything the department wants you to know about. There is a "course information" section of the Math Department page that can also be used to find material on individual courses. While the *URL* of individual pages may change, the department page should always be easy to find. Since it contains features like the department *special permission* form, you should become familiar with it and visit it often (and if you have a suggestion for improving it, you can tell me since I have some responsibility in department computer matters).

This segment of the course. This part of the course will deal with infinite sequences and series, with a brief reappearance of improper integrals. It ends with an exam on Friday, October 22. If time gets too tight, some topics may be deferred until the next segment of the course. Any such changes will be noted on the course Web Page.

Sequences and series. Any process that that produces a first value, then a second, then a third, and so on, is said to define a **sequence**. The terms of the sequence are often denoted a_n, b_n, or something similar, with n taking positive integer values. Some examples also allow an a_0 to be defined. If it is available, we can use it; if not, we won't miss it. When a formula for the terms of the series is known, a_n can be given as a function of n. More interesting ways of obtaining sequences use a *recursive definition*. Here, a_0 or a_1 is a given number, but all others are described by a formula of the form

$$a_{n+1} = f(a_n).$$

An important special case is a **series** S_n, for $n \geq 0$, produced by adding the terms of some sequence a_n, for $n > 0$ (note that we want to have an S_0, although the sequence a begins with a_1). Here we define $S_0 = 0$ since it will represent an empty sum, and define

$$S_{n+1} = S_n + a_{n+1}.$$

It is of no particular importance where a sequence begins since it is easy to change the indexing of a sequence.

Convergence. The central question about any sequence is whether its values, from some point on, are all close to a definite value. That is, does something that will be called

$$\lim_{n\to\infty} a_n$$

exist? The convergence question can ignore any finite number of terms of the sequence, and is not sensitive to changes in the way the sequence is indexed — as long as all terms from some point on are accounted for.

Power series. Many functions behave as if they were polynomials with infinitely many terms. A familiar example is the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n. \tag{G}$$

If $|x| < 1$, the difference between left side of (G) and the sum of the first N terms on the right can be found and shown to approach 0 for large N. The convergence of this series is the basis for many of our tests used to decide on the convergence of series.

The return of the improper integral. Tests based on comparison with a geometric series are inconclusive when applied to series whose n^{th} term behaves like a power of n. For these series, it is possible to compare the series with an improper integral. The limit needed to interpret the improper integral is very similar to that used in studying sequences, and this connection will be used. This suggests that the sequence of partial sums is related to the terms of a series in the same way that an integral is related to the integrand. Extra care is needed because the words “sequence” and “series” are not as different in ordinary language as they must be here as a consequence of the definitions we have given.

Monotonic limits. If the terms of a sequence increase with n, for example, if the sequence is obtained from a series of positive terms, the sequence either converges or “diverges to infinity”. Other sequences can do stranger things, like alternate between two different values, that we consider to be “divergence” because they fail to meet our strict requirement for “convergence”. Our belief in the existence of limits of bounded increasing sequences is essentially the same as saying that real numbers can be expressed as nonterminating decimals, with every process for producing the sequence of digits defining a number.

This theory helps to improve our understanding of the nature of real numbers. One practical consequence of this is that we will get formulas that can be used to approximate values of our favorite functions to any desired accuracy. Such formulas form the basis of the methods actually used by your calculator to give reliable function values quickly.

Lecture topics and homework. The schedule of lectures through the exam of October 22 is given below. Some topics may need to be deferred to allow all *interesting* topics to appear. Conventions on assigned problems are the same as on the first handout.

Date	Section	Page	Discussion Problems	**Hand-in**
October 01	10.1	586	16, 22, 30	**26**
	10.2	596	12, 16, 20, 26	**22**
	10.3	603	2, 8, 10, 14	**16**
October 05	10.4	608	2, 12, 18, 28	**16**
	10.6	619	4, 8, 26	**20**
October 08	10.5	613	8, 10, 14	**18**
	10.7	622	2, 8, 10, 24, 32	**16**
October 12	10.8	627	6, 8, 16, 20	**10**
	10.9	632	6, 10, 16, 20	**2**
October 15	10.10	643	20, 26, 36	**34**
	10.11	647	4, 18	**16**
October 19	10.12	653	4, 26	**6**