

Here is a transcript of a Maple calculation of an example not completed in lecture. To describe the curve it suffices to give  $x$  and  $y$  as expressions in  $t$ .

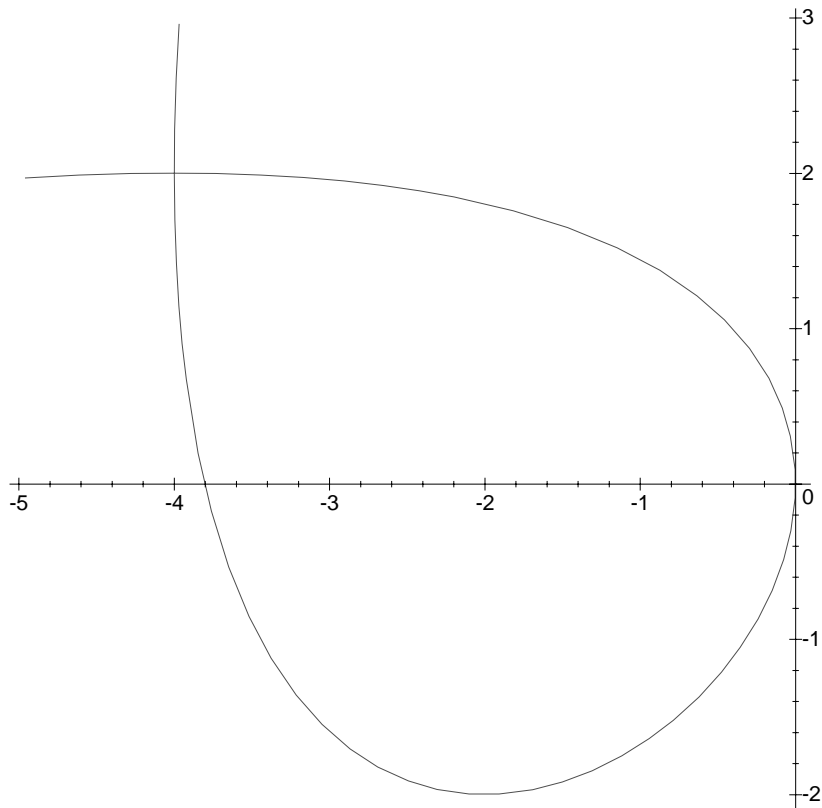
```
> x:=t^3-3*t^2;y:=t^3-3*t;
```

$$x := t^3 - 3t^2$$

$$y := t^3 - 3t$$

This is what the curve looks like. The interval was chosen to include a little more than the points, which will be calculated below, where there are horizontal or vertical tangents.

```
> plot([x,y,t=-1.1..2.1]);
```



We introduce expressions for the derivatives with respect to  $t$ .

```
> x1:=diff(x,t);y1:=diff(y,t);
```

$$x1 := 3t^2 - 6t$$

$$y1 := 3t^2 - 3$$

```
> solve(x1,t);#vertical tangents at these t
```

0, 2

```
> solve(y1,t);#horizontal tangents at these t
```

1, -1

[ This lists the [x,y] values at t = -1,0,1,2 -- all of which are interesting because of the two previous lines.

```
[ > seq([x,y],t=-1..2);
```

[ [-4, 2], [0, 0], [-2, -2], [-4, 2]

[ You should check that these points look right in your picture.

```
[ > dA:=expand(y*x1);#y dx/dt
```

$$dA := 3t^5 - 6t^4 - 9t^3 + 18t^2$$

[ This is the integrand when the usual area integral is expressed in terms of t. It should be integrated left to right on the top and right to left along the bottom. The sum of these will be area as explained in lecture. A single integral from t=-1 to t=2 gives this result. For no good reason the indefinite integral and its values at the endpoint are shown. You can use this to check your calculation of the integral.

[ We then give the single command for finding the integral.

```
[ > A:=int(dA,t);#indefinite integral
```

$$A := \frac{1}{2}t^6 - \frac{6}{5}t^5 - \frac{9}{4}t^4 + 6t^3$$

```
[ > subs(t=2,A);#at upper endpoint
```

$$\frac{28}{5}$$

```
[ > subs(t=-1,A);#at lower endpoint
```

$$\frac{-131}{20}$$

```
[ > "-" ;#difference
```

$$\frac{243}{20}$$

```
[ > Area:=int(dA,t=-1..2);#shortcut to the answer, letting the program do all the work.
```

$$Area := \frac{243}{20}$$

```
[ > evalf(Area);#numerical form, as a decimal
```

[ 12.15000000

[ This says that the loop occupies a little more than 3/4 of the square  $-4 < x < 0$ ,  $-2 < y < 2$ . That looks reasonable.