Homework schedule. Sections through 10.6 should have been done by Oct. 14. Problems through section 10.10 are due on Oct. 21. The exam on Oct. 22 will be based on Sections 10.1 through 10.10. Remaining sections of chapter 10 will be tested in third exam, based on today's lecture. Problems from these sections will be due on Oct. 28.

This week's workshop is a "Practice Exam".

Exercises on the binomial series. The series expansion of $(1+x)^k$ is described in the text, but exercises ask you to extend this to some related expressions. It is expected that simple substitutions, multiplication by powers of x, or identification of constant factors will be done to extend the scope of this formula. If the power is rational, a useful rewriting of the terms of the series may also be possible. Consider

$$\frac{1}{(1+2x)^4} \qquad \#3$$

$$\frac{x}{\sqrt{1-x}} \qquad \#5$$

$$\frac{1}{\sqrt[3]{8+x}} \qquad \#11$$

Section 10.12. This section is called, "applications of the Taylor polynomial". The main contribution is the use of the error term in Taylor's formula to determine whether an approximation is useful. In Section 2.9, you were given the approximate formula

$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$$

as a way to extend your knowledge of a function at a point x to nearby points. The existence of the derivative guarantees that the difference between these two quantities is much smaller than Δx if Δx is small. You were given some things to calculate, but told not to look too closely at the results, although you were assured that they were easily computed approximations to something that is difficult to compute exactly. The use of quadratic approximations was also suggested, since it *looked* better than this linear approximation. With Taylor's formula in our toolkit, we are now in a position to determine when these approximations are likely to be useful.

There is a slight change in notation: x has become a and Δx has become $x-a$, using the general notation of Taylor's theorem. The error formula also introduces a quantity ξ, which is only guaranteed to be between a and x. However, this information is good enough to show that the difference between a function and its n^{th} degree approximation is less than a fixed multiple of $|x-a|^{n+1}$ for a large interval of values of x (centered at a). If your definition of "useful" is "full calculator accuracy", that is, 10^{-12} times the size of $f(a)$ (it is customary to use such a *relative* accuracy rather than an absolute one, since numbers are usually scaled to show a fixed number of leading significant digits), you can determine an interval and a number of terms that will assure that the Taylor polynomial gives what you want.

By using series for the same f with different values of a, you can be sure that there will be some point a close to the x you are interested in. A variant on this, commonly used with trigonometric functions, is to use identities for the function to reduce all calculations to calculations for x close to zero.

Using Taylor's theorem for different functions. Suppose you want to compute $\arctan(x)$. While the repeated differentiation that appears in Taylor's formula gets very complicated fairly soon, the derivative of $\arctan x$ is *exactly* $1/(1+x^2)$, and this can be found with a reasonable error term by substituting in a geometric series.