

Math 152:10-12 — Fall 1999

TF3 CHM-201

Prof. Bumby

Workshop 1, Textbook Sections 5.5, 6.1 and 6.2

1. Use the substitution $u = x^4 + 4x + 1$ to determine the integral:

$$\int \frac{x^3 + 1}{\sqrt{x^4 + 4x + 1}} dx$$

2. Use suitable substitutions to calculate the integrals:

$$(a) \int \frac{e^x}{1 + e^x} dx \qquad (b) \int \frac{e^x}{1 + e^{2x}} dx$$

3. Use the substitution $u = 1 + \sqrt{x}$ to verify that

$$\int_1^4 \frac{dx}{(1 + \sqrt{x})(\sqrt{x})} = \int_2^3 \frac{2du}{u}.$$

Then calculate the second integral and check the first integral directly using the numerical integration facility of your calculator.

Suggestion: On the TI-82, use fnInt (entry 9 from the MATH MATH menu).
Enter fnInt (1 / ($\sqrt{X}+X$) , X , 1 , 4) or fnInt (($\sqrt{X}+X$)⁻¹ , X , 1 , 4)

4. Sketch the region bounded by: $y = \sin(x)$, $y = 0$, $x = 0$, $x = \pi$.

(a) Calculate the area of this region.

(b) Suppose this region is to be divided into four subregions of equal area using vertical lines. Where should the vertical lines be placed?

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5. Consider the function

$$C(x) = \int_0^x \cos(\pi t^2/2) dt$$

(a) Find the derivative, $C'(x)$, the critical points of C in $[0, 2]$, and the intervals in $[0, 2]$ on which C is increasing.

(b) Find $C''(x)$ and the inflection points of C in $[0, 2]$.

(c) Using a calculator or otherwise, sketch the graph $y = C(x)$ in the viewing window $[0, 2] \times [0, 1]$.

(*Suggestion:* On the TI-82, use `fnInt` (entry 9 from the MATH MATH menu).

Set `Y1 = fnInt(cos (pi (T^2)/2), T, 0, X)`).

6. Calculate the integrals and interpret the answer geometrically:

$$(a) \int_0^2 (x^2 - 1) dx \qquad (b) \int_0^2 |x^2 - 1| dx$$

7. Let \mathcal{F} be the figure whose base is a circle of radius 1 in the xy -plane and whose cross-sections perpendicular to the x -axis are equilateral triangles with one side in the xy -plane. (In order to have a reasonable shape, assume that the other vertex of the triangle is always above the base.)

(a) Sketch some cross-sections of \mathcal{F} .

(b) Find the volume of \mathcal{F} .

(c) Try to visualize the solid \mathcal{F} . In particular, what are the cross-sections perpendicular to the y -axis?

8. Find the volume of the figure obtained by rotating a circle of radius r with center at $(R, 0)$ about the y -axis, where $0 < r < R$. (This is essentially Exercise 6.2.63 of the textbook. You may find the picture on page 397 helpful.)

End workshop 1