Math 152:10–12 — Fall 1999
TF3 CHM-201
Prof. Bumby

Workshop 2, Textbook Chapter 6 and Section 7.1

1. **(a)** Sketch the first quadrant region $\mathcal{A}$ bounded by the parabola $y = x^2$, the tangent to the parabola at $(1, 1)$, and the x-axis.

(b) Find the area of $\mathcal{A}$.

(c) Set up an integral giving the volume of the figure obtained by rotating $\mathcal{A}$ about the y-axis using the method of washers.

(d) Set up an integral giving the volume of the figure obtained by rotating $\mathcal{A}$ about the y-axis using the method of cylindrical shells.

(e) Evaluate one of the integrals in (c) or (d) to find this volume.

(f) Find the volume of the figure obtained by rotating $\mathcal{A}$ about the x-axis. You may use any method.

2. Sketch the graph of $\mathbf{C}_1$ with equation $y = \sin x$ and $\mathbf{C}_2$ with equation $y = \cos x$ in the viewing window $[0, 2\pi] \times [-1, 1]$. Use your sketch to identify the region $\mathcal{B}$ in this window that is below $\mathbf{C}_1$ and above $\mathbf{C}_2$, with parts of these curves forming the entire boundary of $\mathcal{B}$. Then find the area of $\mathcal{B}$.

3. A solid in three-dimensional space has its base in the first quadrant of the xy-plane. The base is bounded by the curves:

$$y = x^{1/3} \text{ and } y = x^2.$$

Find the volume of the solid if:

(a) a plane perpendicular to the x-axis slices through the solid in a square with one side on the base;

(b) a plane perpendicular to the y-axis slices through the solid in an equilateral triangle with one side on the base.

. . . continued on other side

4. In Section 5 of the *Review and preview* chapter (page 41), average velocity was defined as

$$\frac{\text{distance traveled}}{\text{time elapsed}}.$$

This was used in Section 1.1 to motivate a definition of *instantaneous velocity* as the derivative of distance with respect to time. Thereafter, the word "velocity" always referred to instantaneous velocity. We now have another meaning of the word "average", introduced in Section 6.5 (page 408). Show that the old meaning of average velocity is the average of a function giving velocity as a function of time.

If the velocity is always of the same sign, the function giving distance as a function of time has an inverse, so all properties of the motion can be expressed as a function of distance. It is thus possible to consider averaging velocity with respect to distance. These two averages can be different. Show that this happens when the motion is given by *free fall*, with displacement given by $s = \left(1/2\right) g t^2$. (See exercise 6.5.16, page 410).

5. **(a)** Obtain the reduction formula

$$\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx \qquad (*)$$

using integration by parts with $u = (\ln x)^n$ and $dv = dx$.

(b) Use the substitution $x = e^y$ to replace $(*)$ with a formula involving functions of y.

(c) Give a direct proof of the formula obtained in (b) using integration by parts.

End workshop 2