Math 152:10–12 — Fall 1999
TF3 CHM-201
Prof. Bumby

Workshop 6, Textbook Sections 10.5 through 10.8

1. For a positive, decreasing sequence

$$\{a_n\}_{n=1}^{\infty}$$

with

$$\lim_{n\to\infty} a_n = 0,$$

the alternating series test shows that

$$\left|\sum_{N+1}^{\infty}(-1)^n a_n\right| \le a_{N+1}.$$

(a) How large does N have to be to insure that

$$\sum_{n=1}^{N}\frac{(-1)^n}{n^3} \text{ is within } 10^{-8} \text{ of } \sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}?$$

(b) How large does N have to be to insure that

$$\sum_{n=1}^{N}\frac{(-1)^n}{n^2(n+1)^2} \text{ is within } 10^{-8} \text{ of } \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2(n+1)^2}?$$

(c) Calculate the sum of one of the series (a) or (b) with error of at most $1/10^8$.

2. Sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are defined by:

$$a_n = \begin{cases} 1/n^2, & \text{if } n \text{ is not a perfect square;} \\ 1/\sqrt{n}, & \text{if } n \text{ is a perfect square.} \end{cases}$$

$$b_n = \begin{cases} 1/n^2, & \text{if } n \text{ is not a power of 2;} \\ 1/\sqrt{n}, & \text{if } n \text{ is a power of 2.} \end{cases}$$

Which, if any, of the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ is convergent or divergent? Explain your answer.

. . . continued on other side

3. Under the hypothesis of the integral test, if $a_n = f(n)$ and $s_n = a_1 + a_2 + \cdots + a_n$, then

$$\int_1^n f(x)\,dx \le s_n \le a_1 + \int_1^n f(x)\,dx$$

In the case of the harmonic series $\sum 1/n$, this gives the inequality

$$\ln(n) \le 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \le \ln(n) + 1$$

(a) Find the analogous inequalities for s_n for the divergent series:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \qquad \text{and} \qquad \sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

(b) Estimate the sum of the first n terms of each of the three series, to within 1, if $n = 10^{10}$ or if $n = 10^{100}$.

(c) Of the three series, which diverges most rapidly and which diverges least rapidly?

4. Use the geometric series to express each of the series as a rational function, i.e., $f(x)/g(x)$ with $f(x)$, $g(x)$ polynomials. Also find for which values of x each series converges.

$$(a)\ \sum_{n=0}^{\infty} x^{3n+2} \qquad (b)\ \sum_{n=0}^{\infty} \frac{x^{2n}}{9^n} \qquad (c)\ \sum_{n=0}^{\infty} \frac{x^{2n}}{3^n} \qquad (d)\ \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

5. Find the radius and interval of convergence of the following power series. At the end points of the interval of convergence, determine if the series is conditionally convergent, absolutely convergent, or divergent.

$$(a)\sum_{n=0}^{\infty} \frac{nx^n}{n^2+1} \qquad (b)\sum_{n=1}^{\infty} \frac{n^2(x-1)^n}{2^n} \qquad (c)\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$

End workshop 6