Math 152:10–12 — Fall 1999
TF3 CHM-201
Prof. Bumby

Workshop 10, Taylor's formula and the use of graphing calculators.

1. Consider the function

$$f(x) = (1-x)^{-1/2} = \frac{1}{\sqrt{1-x}}.$$

(a) Find the fourth Taylor polynomial, $T_4(x)$, centered at $a = 0$ of $f(x)$. You may use either Taylor's theorem from Section 10.10 or the binomial series from Section 10.11.

(b) Sketch the graphs of $y = f(x)$ and $y = T_4(x)$ in the window $[-1, 1] \times [0, 3]$.

(c) Sketch the graph of $y = f(x) - T_4(x)$ in the window $[-.5, 5] \times [-.01, .01]$.

(d) Give an upper bound you can give for $|f(x) - T_4(x)|$ on the interval $[-.5, .5]$? You should not necessarily try to get the *best* estimate on this error, but your answer should be an explicit number valid for every x on this interval. The Lagrange form of the error for Taylor series gives a convenient estimate, but once the series is known to converge to $f(x)$, you may have some other way to estimate the sum of the remaining terms. When you are well within the interval of convergence, the series is converging like a geometric series so an upper bound on the error is the fifth degree term of the series divided by $(1-r)$ where r is an upper bound on the ratio of the absolute value of $(n+1)^{\text{st}}$ degree term to the n^{th} degree term for $n \geq 5$.

2. The linear approximation for the function $f(x) = x^5$ near $x = 2$ is $32 + 80(x-2)$. (You should check this!)

(a) What coefficient a should be chosen to give the best quadratic approximation

$$x^5 \approx 32 + 80(x-2) + a(x-2)^2 \qquad \text{near } x = 2?$$

(b) If this approximation is used for various x's in the interval $[2, 2.1]$, can you be certain that the error is no bigger than .05? Explain, using Lagrange's formula for the remainder.

(c) Graph $x^5 - 32 - 80(x-2) - a(x-2)^2$ (using the value of a found in part a)) in the interval $[2, 2.1]$.

. . . continued on other side

The remainder of this workshop suggests experiments on using the graphing calculator in its parametric and polar equation modes.

3. If $x = \cos\theta$ and $y = \sin\theta$, then $x^2 + y^2 = 1$, so points with this description lie on the circle of radius 1 centered at the origin.

(a) If you use the interval $0 \le \theta \le \pi$, how much of the circle do you get?

(b) If you use the interval $-\pi/2 \le \theta \le \pi/2$, how much of the circle do you get?

(c) If you use the interval $0 \le \theta \le 2\pi$, how much of the circle do you get?

(d) If you use the interval $-\pi \le \theta \le \pi$, how much of the circle do you get?

(e) What difference would you see if you used a slightly smaller upper endpoint in (c) or (d)?

4. In polar coordinates, $r = 1$ is a description of the circle of radius 1 with center at the origin. However, this is not the only example of a circle with a nice equation in polar coordinates. The rectangular equation $x^2 + y^2 = 2x$ is easily seen to give a circle with center $(1, 0)$ and radius 1, so that it passes through the origin (note that $x = 0$ and $y = 0$ satisfies the equation). To express this in polar coordinates, use $x^2 + y^2 = r^2$ and $x = r\cos\theta$ to get $r^2 = 2r\cos\theta$, and then divide by r to get $r = 2\cos\theta$.

(a) Plot $r = 1$, experimenting with different choices of interval for θ. (For example, you can use the suggestions in problem 3.)

(b) Plot $r = 2\cos\theta$ for $-\pi/2 \le \theta \le \pi/2$. (These are two consecutive values of θ that lead to $r = 0$.)

(c) Plot $r = 2\cos\theta$ for $0 \le \theta \le \pi/2$, and identify which point on the graph corresponds to which end of this interval of values of θ.

(d) Plot $r = 2\cos\theta$ for $0 \le \theta \le 2\pi$. Do you see any difference between this and the result of (b)?

(e) Combine the plots of (a) and (b) on the same axes and try to identify the points of intersection from the graph. Then use the equations of the curves to locate these points exactly.