

Apr. 7, 1999

1. Fill in: The endpoints of the interval of convergence of the of the power series  $\sum_{n=0}^{\infty} \frac{3^n(x-1)^n}{n^2-5}$  are at  $x = \underline{\hspace{1cm}}$  and  $\underline{\hspace{1cm}}$ . (Do not worry about whether the series converges at the endpoints.) **SOLUTION:** The endpoints are  $x = \frac{2}{3}$  and  $x = \frac{4}{3}$ . To see this use the ratio test.

$$\lim_{n \rightarrow \infty} \left| \frac{3^{n+1}(x-1)^{n+1}(n^2-5)}{3^n(x-1)^n((n+1)^2-5)} \right| = \lim_{n \rightarrow \infty} \left| \frac{3(x-1)(n^2-5)}{(n+1)^2-5} \right| = 3|x-1|.$$

The series converges absolutely for  $3|x-1| < 1$  and diverges for  $3|x-1| > 1$ . The endpoints of the interval of convergence are at  $3|x-1| = 1$ , that is,  $x = \frac{2}{3}$  and  $x = \frac{4}{3}$ .

2. Find the second-degree Taylor polynomial  $T_2(x)$  for the function  $f(x) = x^{-5}$  about  $x = -1$ . **SOLUTION:**  $f'(x) = -5x^{-6}$  and  $f''(x) = 30x^{-7}$ , so  $f(-1) = -1$ ,  $f'(-1) = -5$  and  $f''(-1) = -30$ . The second-degree Taylor polynomial is

$$T_2(x) = - \left( 1 + 5(x+1) + \frac{30}{2!}(x+1)^2 \right).$$

3. Use power series that you know to express  $\int_0^{0.3} e^{x^3} dx$  as an infinite series. (Do not attempt to evaluate or approximate the sum of that series.) **SO-**

**LUTION:**  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$  for all real numbers  $x$ , so  $e^{x^3} = \sum_{n=0}^{\infty} \frac{(x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$

for all  $x$ . Then

$$\int_0^{0.3} e^{x^3} dx = \int_0^{0.3} \left( \sum_{n=0}^{\infty} \frac{x^{3n}}{n!} \right) dx = \sum_{n=0}^{\infty} \left[ \int_0^{0.3} \frac{x^{3n}}{n!} dx \right] = \sum_{n=0}^{\infty} \frac{(0.3)^{3n+1}}{(3n+1)n!}.$$