Problems for Section 6.2

(1) The base of a solid is the region enclosed by the ellipse $4x^2 + 16y^2 = 1$ in the xy-plane. Any cross-section of the solid perpendicular to the x-axis is a rectangle with a longer side on the base and shorter sides that have length equal to half of the length of the longer sides. Find the volume of the solid.

(2) Consider the bounded region in the xy-plane enclosed by $y = x^4$ and $y = x^5$. (a) Find the volume of the solid that results when this region is rotated about the x-axis. (b) Find the volume of the solid that results when this region is rotated about the y-axis.

(3) Consider the region in the xy-plane which is enclosed by $y = \sin x$, $y = 0$ between $x = 0$ and $x = \pi$. Find the volume of the solid that results when this region is rotated about the line $y = -2$. Hint: Use a trigonometric identity for $\sin^2 x$.

(4) Use the method of washers to find the volume of the solid obtained when the circle $x^2 + (y-3)^2 = 1$ is rotated about the x-axis. Hint: Evaluate $\int_{-1}^{1} \sqrt{1-x^2}\, dx$ using plane geometry.

(5) Use the method of disks to find the volume of a sphere of radius 2.

Problems for Section 6.3

(1) Use the method of shells to find the volume of the solid obtained when the circle $(x-3)^2 + y^2 = 1$ is rotated about the y-axis. Hints: Substitute $u = x - 3$. Evaluate $\int_{-1}^{1} \sqrt{1-u^2}\, du$ using plane geometry. Note that the solid in this problem has the same size and shape (hence the same volume) as the solid in problem (4) of section 6.2.

(2) Find the volume of the solid that results when the region bounded by $y = 4$, $x = 0$ and $y = 2x$ is rotated about the y-axis.

(3) Find the volume of the solid that results when the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$ is rotated about the line $x = 2$.

(4) Use the method of shells to find the volume of a sphere of radius 2.

(5) Find the volume of the solid that results when the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 1$ and $x = 2$ is rotated about the y-axis.

(6) Find the volume of the solid that results when the region bounded by $y = x^2$ and $y = 5x - 6$ is rotated about the y-axis.

(7) Find the volume of the solid that results when the region bounded by $y = x^2$ and $y = 5x - 6$ is rotated about the x-axis.

Problems for Section 6.4

(1) When a particle is at position x on the positive x-axis, it experiences a force of $\sqrt{x}$ pounds from the right. Assume that distances on the x-axis are measured in feet. Find the work done when the particle is pushed from $x = 4$ to $x = 9$.

(2) A 3 foot long chain is lying on a closet floor. Its weight density is 2 pounds per foot. How much work is required to pick up the chain by the link at one end, and to hook this link on a wall nail that is located 4 feet above the floor? Assume that the links have negligible thickness. Note that the link at the other end is 1 foot above the floor when the chain is hooked this way.

(3) We have a conical water tank standing on its vertex. Its height is 10 feet. Its radius at the top is 2 feet. It is filled with water up to the 7 foot height. How much work is required to drain this water through a spigot at the top of the tank? Assume that the density of water is 62.5 pounds per cubic foot.

Problems for Section 6.5

(1) Find the average value of $f(x) = \sqrt{x+4}$ over the interval $[-1, 3]$.

(2) Assume $f(x) = x^2 - 4x + 10$. Find both values of c in the interval $[1, 3]$ which have the property $f(c) =$ the average of $f(x)$ over $[1, 3]$.

(3) A car passed the 20 mile marker on a highway at 1:00 pm. It passed the 140 mile marker at 5:00 pm. Use the Fundamental Theorem of Calculus and the Mean Value Theorem for Integrals to show that the car's speed must have been exactly 30 miles per hour at some instant. Draw the same conclusion using the ordinary Mean Value Theorem in section 4.2.