## Problems for Section 11.1

(1) Find $\lim_{n\to\infty} \dfrac{2n^2+3n+5}{5n^2+n+2}$ .

(2) Find $\lim_{n\to\infty} \dfrac{\ln n}{\sqrt{n}}$ .

(3) Assume $a_1 = 4$, $a_{n+1} = \frac{1}{3}(a_n + 5)$ for $n = 1, 2, 3, \ldots$ . Assume also that $\lim_{n\to\infty} a_n$ exists. Find $\lim_{n\to\infty} a_n$

(4) Find $\lim_{n\to\infty} n^2 \sin\left(\dfrac{1}{n^2}\right)$ .

(5) Find $\lim_{n\to\infty} \left(1 - \dfrac{7}{n}\right)^n$ .

## Problems for Section 11.2

(1) Suppose that we have a sequence $a_1, a_2, a_3, \ldots$ such that its $n$th partial sum is $4 - 3^{-n}$. Give an explicit formula for $a_n$. Does $\sum_{n=1}^{\infty} a_n$ converge or diverge? Explain. If the series converges, then find the sum.

(2) Express the repeated decimal $3.474747\ldots$ as a ratio of two natural numbers. The natural numbers are $1, 2, 3, \ldots$ .

(3) Find the sum of the series $\displaystyle\sum_{n=3}^{\infty} \frac{3^n - 2^n}{2^{2n}}$ .

(4) Find the sum of the telescoping series $\displaystyle\sum_{n=5}^{\infty} \frac{2n+1}{n^2(n+1)^2}$. Hint: Look at page 717 and rewrite the series so that only expressions of the form $1/k^2$ are used.

(5) Does $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{8n^2 - n - 2}{5n^2 + 7n + 10}}$ converge or diverge? Explain. Does $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+3} n^{1/n}$ converge or diverge? Explain.

(6) Assume that $a_n$ and $b_n$ are sequences for which we know $\sum_{n=1}^{\infty} a_n = 7$, $\sum_{n=1}^{\infty} b_n = 5$, $a_1 = 3$, $a_2 = -1$, $b_1 = -8$, $b_2 = 4$. Find the sum $\sum_{n=3}^{\infty}(2a_n - b_n)$ (which begins with $n = 3$).

## Problems for Section 11.3

(1) Test the series $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$ for convergence or divergence.

(2) Test the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$ for convergence or divergence.

(3) Use the integral test to determine whether $\sum_{n=1}^{\infty} \frac{n}{e^n}$ converges or diverges.

(4) Test the series $\sum_{n=10}^{\infty} \frac{1}{n(\ln n)(\ln(\ln n))}$ for convergence or divergence.

(5) Find a natural number $N$ such that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{N} \frac{1}{n^2}$ differ by less than $10^{-6}$.

(6) Suppose that we approximate $\sum_{n=1}^{\infty} \frac{1}{n^2}$ by $\sum_{n=1}^{1,000} \frac{1}{n^2}$. Find numbers $A$ and $B$ such that $A \leq \text{error} \leq B$ and $B - A < 10^{-6}$.

## Problems for Section 11.4

(1) Determine whether $\sum_{n=5}^{\infty} \sqrt{\frac{2n^2+n+1}{n^5-n^2-3}}$ converges or diverges.

(2) Determine whether $\sum_{n=1}^{\infty} \sqrt{\frac{2n^2-n-1}{5n^4+n^3+10}}$ converges or diverges.

(3) Determine whether $\sum_{n=0}^{\infty} \frac{1+\cos(n^3+e^n)}{2^n}$ converges or diverges.

(4) Determine whether $\sum_{n=1}^{\infty} \sin(1/n^{3/2})$ converges or diverges.

(5) Determine whether $\sum_{n=1}^{\infty} \frac{2^n+20}{3^n-n}$ converges or diverges.

(6) Determine whether $\sum_{n=1}^{\infty} \frac{\ln n}{n^{5/2}}$ converges or diverges. Hint: Show $\lim_{n\to\infty} \frac{\ln n}{n} = 0$.

(7) Determine whether $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ converges or diverges. Hint: Show $\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = 0$.

**Problems for Section 11.5**

(1) Determine whether $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{2n^2+5}$ converges or diverges.

(2) Determine whether $\sum_{n=5}^{\infty} (-1)^n \frac{1}{\sqrt{\ln n}}$ converges or diverges.

(3) Determine whether $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$ converges or diverges.

(4) Determine whether $\sum_{n=5}^{\infty} (-1)^n (n^{1/n} - 1)$ converges or diverges.

(5) How many terms of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ do we need to add up in order to approximate the infinite sum with an accuracy better than $10^{-6}$?

(6) How many terms of $\sum_{n=1}^{\infty} \frac{(-1)^n}{8n+1}$ do we need to add up in order to approximate the infinite sum with an accuracy better than $10^{-6}$?

**Problems for Section 11.6**

(1) Determine whether $\sum_{n=5}^{\infty} \frac{n!(2n)!5^n}{(3n)!}$ converges or diverges.

(2) Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!3^n}$ converges or diverges.

(3) Determine whether $\sum_{n=1}^{\infty} \frac{\cos(n^4+2)}{n^{5/4}}$ converges or diverges.

(4) Determine whether $\sum_{n=1}^{\infty} \frac{n!2^n}{n^n}$ converges or diverges.

(5) Determine whether $\sum_{n=1}^{\infty} \frac{(n^3+n+8)\sin(e^n)}{2^n}$ converges or diverges.

(6) Determine whether $\sum_{n=1}^{\infty} \frac{2^n + n}{3^n - 1}$ converges or diverges.

(7) Determine whether $\sum_{n=1}^{\infty} \left( \frac{2n^2 + n + 15}{3n^2 - n - 1} \right)^n$ converges or diverges.

(8) Assume that the numbers $a_n$ are defined recursively by $a_1 = 27$, $a_{n+1} = -\frac{3n+1}{4n+3} a_n$ for $n = 1, 2, 3, \ldots$. Determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges.

## Problems for Section 11.7

(1) Determine whether $\sum_{n=5}^{\infty} \frac{(4n^3 + n^2 + 5)\sin(n^2 - n)}{n^5 - n - 1}$ converges or diverges.

(2) Determine whether $\sum_{n=6}^{\infty} \frac{n^3 - n - 7}{7n^4 + n^3 + 2}$ converges or diverges.

(3) Determine whether $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$ converges or diverges.

(4) Determine whether $\sum_{n=8}^{\infty} \frac{(n+10)n! \cos(2^n - 3^n)}{(n+3)!}$ converges or diverges.

(5) Determine whether $\sum_{n=5}^{\infty} \frac{\cos(n\pi) n^3}{n^4 - 2}$ converges or diverges.

(6) Determine whether $\sum_{n=2}^{\infty} \left( \frac{n}{n + 0.5} \right)^{n^2}$ converges or diverges.
Hint: What is $\lim_{n \to \infty} \left( 1 + \frac{1/2}{n} \right)^n$?

(7) Determine whether $\sum_{n=9}^{\infty} \frac{n^5 4^n}{5^n}$ converges or diverges.

(8) Determine whether $\sum_{n=6}^{\infty} \frac{\cos(2^n + 3)\sqrt{n^2 - 4}}{n^3 + n + 7}$ converges or diverges.

(9) Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{4n^2 + 1}}$ converges or diverges.

(10) Determine whether $\sum_{n=3}^{\infty} \frac{(n!)^2 3^n}{(2n)!}$ converges or diverges.

(11) Determine whether $\sum_{n=2}^{\infty} \frac{3^n + n^2}{5^n - n^2}$ converges or diverges.

## Problems for Section 11.8

(1) Find the interval of convergence of $\sum_{n=3}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ .

(2) Find the interval of convergence of $\sum_{n=2}^{\infty} \frac{2^n (x-3)^{2n}}{n^{3/2}}$ .

(3) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{n^2 (x+4)^n}{3^n}$ .

(4) Find the interval of convergence of $\sum_{n=3}^{\infty} \frac{(-5)^n (x-1)^n}{n}$ .

(5) Find the radius of convergence of $\sum_{n=5}^{\infty} \left[ \left(1 + \frac{1}{n}\right)^{n^2} x^n \right]$ using the Root Test.

(6) Assume that $\sum_{n=0}^{\infty} c_n (x-7)^n$ diverges when $x = 2$ and converges when $x = 3$. Does it converge or diverge when $x = 10$? Explain. Does it converge or diverge when $x = 15$? Explain.

(7) Is it possible to have a power series $\sum_{n=0}^{\infty} c_n (x-7)^n$ which converges when $x = 2$ and diverges when $x = 3$? Explain.

## Problems for Section 11.9

(1) Find the power series centered at 0 that represents $f(x) = \frac{1}{1+x^4}$. Determine the interval of convergence of this power series.

(2) Find the power series centered at 0 that represents $f(x) = \frac{1}{25+x^2}$.

(3) Find the power series centered at 0 that represents $f(x) = \frac{x^3}{1+25x^2}$.

(4) Find the power series centered at 0 that represents $f(x) = \dfrac{1}{(x-2)(x-3)}$. Hint: Use partial fractions.

(5) Find the power series centered at 0 that represents $f(x) = \dfrac{x^{10}}{(4+x)^2}$. Hint: Use a function $g(x)$ such that $g'(x) = \dfrac{1}{(4+x)^2}$, and later multiply by $x^{10}$.

(6) Express $\displaystyle\int \frac{x - x^3/3 - \tan^{-1} x}{x^5}\, dx$ as a power series.

(7) Use a power series to approximate $\displaystyle\int_0^{1/2} \ln(1+x^2)\, dx$ with an accuracy better than $10^{-5}$.

**Problems for Section 11.10**

(1) Find the 3rd-degree Taylor polynomial $T_3(x)$ of $f(x) = \sin x$ centered at $a = \pi/6$.

(2) Find the Maclaurin series of $\cos^2(x/4)$ using the Maclaurin series of $\cos x$ and a trigonometric identity.

(3) Find a reasonable estimate for $|\sin(10^{-2}) - (10^{-2} - 10^{-6}/6)|$. Hint: $10^{-2} - 10^{-6}/6 = T_4(10^{-2})$.

(4) Find the Maclaurin series for $f(x) = \dfrac{e^{2x} - 1 - 2x}{x^2}$ using the Maclaurin series for $e^x$.

(5) Use $\tan x = x + (1/3)x^3 + (2/15)x^5 + \cdots$ and the Maclaurin series for $\cos x$ to compute the first three nonzero terms in the Maclaurin series for $(\tan x)(\cos x)$. Since $(\tan x)(\cos x) = \sin x$, you know what the answer should be. This is a way to check the arithmetic in the long division that led to $\tan x = x + (1/3)x^3 + (2/15)x^5 + \cdots$.

(6) Find $\displaystyle\lim_{x\to 0} \frac{x + (1/3)x^3 - \tan x}{x^5}$ using a simple method that does not involve l'Hôpital's Rule.

**Problems for Section 11.11**

(1) Find the first four nonzero terms of the Maclaurin series for $\dfrac{1}{(1+x)^{1/3}}$.

(2) Find the first four nonzero terms of the Maclaurin series for $\dfrac{1}{(1+x)^{2/3}}$.

(3) Find the first four nonzero terms of the power series which is the product of the power series in problem (1) and the power series in problem (2). Since $\frac{1}{(1+x)^{1/3}} \cdot \frac{1}{(1+x)^{2/3}} = \frac{1}{1+x}$, you know what the answer should be. This is a way to check the arithmetic in (1) and (2).

(4) Find the first four nonzero terms of the Maclaurin series for $\sqrt{1+x^4}$.

**Problems for Section 11.12**

(1) Find a Taylor polynomial with center 0 that approximates $\cos x$ with an accuracy better than $10^{-16}$ over the interval $-0.1 \le x \le 0.1$.

(2) Find a Taylor polynomial with center 0 that approximates $e^x$ with an accuracy better than $10^{-10}$ over the interval $-0.1 \le x \le 0.1$.

(3) Approximate the definite integral $\int_0^{1/2} e^{-x^4}\, dx$ with an accuracy better than $10^{-9}$. Hint: Use a Maclaurin series.

(4) Find $\lim_{x \to 0} \frac{2x - (4/3)x^3 - \sin(2x)}{\big(3x - \tan(3x)\big)\big(1 - \cos(5x)\big)}$. Do not use l'Hôpital's Rule.