REVIEW PROBLEMS FOR MIDTERM ONE
MATH 151, SPRING 2008

1. Express the following sets as intervals:

$$\{x : \ |x-5| > 4\}, \qquad \{x : \ |5x+3| \le 2\}.$$

2. Let $f(x) = |x|$. Sketch the following graphs:
(a) $f(x)$
(b) $f(x) - 3$
(c) $f(x+2)$
(d) $\frac{1}{10} f(x-1) + 4$

3. Find the domain for the following functions:

$$f(x) = \sqrt{x+1}, \qquad g(x) = \frac{2}{3-x}, \qquad h(x) = \sqrt{x^2 - x + 5}.$$

4. Write the equations of the line satisfying the following conditions:
(a) Slope $k = \frac{1}{2}$ and point $(3, -4)$.
(b) The line perpendicular to (a) and pass point $(3, -4)$.

5. Prove that $x^2 + 3x + 3 \ge 0$ for all x.

6. Solve $\sin 2x + \cos x = 0$ for $0 \le x \le 2\pi$.

7. Find the inverse of $f(x) = \frac{x-2}{x-1}$ and determine its domain and range.

8. Evaluate the limit if possible or state it does not exist.

$$\lim_{x \to 4} (3 + \sqrt{x}), \qquad \lim_{x \to -1} \frac{3x^2 + 4x + 1}{x+1}, \qquad \lim_{t \to 9} \frac{\sqrt{t} - 3}{t - 9}.$$

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3}, \qquad \lim_{\theta \to 0} \frac{\sin 5\theta}{\theta}, \qquad \lim_{t \to 0} \frac{\sin 2t}{\sin 3t}.$$

$$\lim_{x \to 1^+} \left(\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{x^2-1}} \right), \qquad \lim_{s \to 0} \frac{\cos s - 2}{s}.$$

9. Graph $h(x)$ and describe the type of discontinuity, $h(x) = \begin{cases} e^x & x \le 0 \\ \ln x & x > 0 \end{cases}$.

10. Find the

11. Suppose that f is a differentiable function with $f(1) = 2, f'(1) = 4$ and $g(x) = x^4 - x + 1$. Use this information to calculate:

(a) $(fg)'(1)$. (b) $(f/g)'(1)$. (c) $f \circ g)(0)$. (d) $(f \circ g)'(0)$. (e) $(f \circ g)'(1)$.

12. With A and B constants, a function f is defined by $f(x) = \begin{cases} \frac{1}{x} + A & x < -1 \\ |x| & -1 \leq x \leq 1 \\ \frac{1}{x} + B & 1 < x \end{cases}$.

(a) Find A and B so that f is continuous everywhere.
(b) Sketch the graph of $f(x)$.
(c) At which points does f fail to be differentiable?

13. Let $f(x) = 2\sin x + \sin^2 x$.
(a) Find $f'(x)$.
(b) Find an equation of the tangent line to the graph of $f(x)$ at $x = \pi$.
(c) Find all values of x for which the tangent line is horizontal.

14. Find the derivative of the following functions:

(a) $f(x) = \ln(x + e^x)$. (b) $f(x) = \dfrac{e^{x^2}}{1 + \sin 2x}$. (c) $h(y) = (y^2 + \sin(2y))\cos(2y - 1)$.

(d) $y = \tan(\cos(3x - 1))$. (e) $y = (x + \frac{1}{x})^3$. (f) $y = \sin\sqrt{x^2 + 1}$.

15. Using the formal definition of limit to justify that $\lim_{x \to 2}(2x^2 - x) = 6$.

16. Prove rigorously that $\lim_{x \to 5} \frac{1}{x} = \frac{1}{5}$.

17. Using the Intermediate volume theorem to show that there is $c \in (0, 1)$ such that $e^{c^2} = c$.

18. Find an example of a continuous function which is not differentiable.

19. The height (in feet) of a helicopter at time (in minutes) is $h(t) = -3t^3 + 400t$ for $0 \leq t \leq 10$.
(a) Plot the graphs of $h(t)$ and the velocity $v(t)$.
(b) Find the velocity at $t = 6$ and $t = 7$.
(c) Find the maximum height of the helicopter.

20. Let $s(t) = \dfrac{t^3}{3} - \dfrac{5t^2}{2} + 6t$ be the position function of some moving particle.
(a) Find the velocity function.
(b) Determine where the particle stop.
(c) Find when the particle is moving forward and backsword.

(d) Find the distance the particle travels during the first 4 seconds.

21. Indicate whether each of the following 5 statements is true (T) or false (F). For one statement which you mark as true, add a brief reason. Also, for one statement which you mark as false, add a brief reason.

(a) A vertical line intersects the graph of a function at most once.

(b) The domain of $\tan x$ is all real numbers.

(c) The function $f(x) = x^2$ is defined for all real numbers x, and is a one-to-one function.

(d) If $\underset{x\to 2}{f}(x) = 0$ and $\lim_{x\to 2} g(x) = 0$, then $\lim_{x\to 2} \frac{f(x)}{g(x)}$ does not exist.

(e) If the derivative $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.

22. Imagine a sphere whose radius r increases at a rate of 3 cm/s. At what rate is the volume V of the sphere increasing when $r = 10$ cm?

23. Calculate the first five derivatives of $f(x) = \sqrt{x}$.

24. The position function of a particle moving in a straight line during a 5-s trip is $s(t) = t^2 - t + 10$ cm.

(a) What is the average velocity for the entire trip?

(b) Is there a time at which the instantaneous velocity is equal to this average velocity? Is so, find it.

25. Find all points (a, b) on the parabola $y = x^2 - x$ so that the tangent line to the parabola at the point (a, b) contains point $(2, 1)$.

26. Explain why $x^3 + x - 1$ has a real root in $(0, 1)$.

27. Suppose that u is a positive real number and a, b, c are real numbers so that $log_u(5) = a, log_u(27) = b$ and $log_u(32) = c$. What is the numerical value of $u^{2a+\frac{1}{3}b-\frac{2}{5}c}$?

28 Find equations of the tangent lines to the curve $y = \frac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.

29. Find the derivative of the following functions:

(a) $y = \sqrt{\dfrac{z+1}{z-1}}$ (b) $y = \sin(\cos(\tan x))$ (c) $y = (1 + \cot^5(x^4 + 1))^{-\frac{3}{4}}$

(d) $f(x) = \sin(x^2)\cos^2 x$ (d) $y = \dfrac{\cos\theta}{1 + \sin\theta}$ (e) $h(t) = \dfrac{\sec t}{t^2}$

30. The size of certain animal population $P(t)$ at time t (in months) satisfies $\frac{dP}{dt} = 0.2(300 - P)$. (a) Is P growing or shrinking when $P = 250$? when $P = 350$?

(b) Sketch the graph of $\frac{dP}{dt}$ as a function of P for $) \leq P \leq 3000$.