(10) 1. Find an equation for the function $f(x)$ if $f''(x) = 6x + 4$ and $f(2) = 1$, $f'(2) = -1$.

(15) 2. A differentiable function $y = f(x)$ has the following properties:

(1) The domain is $(-\infty, \infty)$.

(2) The only $x$-intercept is $x = 0$.

(3) $\lim_{x \to \infty} f(x) = -\infty$ and $\lim_{x \to -\infty} f(x) = 0$.

(4) $f(x)$ is negative for $x > 0$ and $f(x)$ is positive for $x < 0$. Moreover, $f(1) = -9$, $f(-2) = 2$, and $f(-1) = 3$.

(5) $f'(x)$ is 0 only at $x = -1$. Moreover, $f'(-4)$ is positive while $f'(4)$ is negative.

(6) $f''(x)$ is 0 only at $x = -2$. Moreover, $f''(-6)$ is positive while $f''(4)$ is negative.

(a) Determine the intervals where the function is increasing and the intervals where it is decreasing.

(b) Determine the $x$-values where the function has a local maximum and those where it has a local minimum.

(c) Determine the intervals where the function is concave up and the intervals where it is concave down.

(d) Determine the $x$-values where the inflection points occur.

(e) Determine all vertical and horizontal asymptotes.

(f) Sketch the graph of a function which has the properties found in parts (a)-(e) above. Show on the graph all intercepts, asymptotes, local maxima and local minima, and points of inflection.
(20) 3. Compute \( \frac{dy}{dx} \) in each of the following cases. You don’t have to simplify.

(a) \( y = \sqrt{3 + \frac{5}{x^9}} \)

(b) \( y = x^5 \tan^{-1}(8 - 7x) \)

(c) \( y = \frac{\cos(5 \ln x)}{7 + e^{8x}} \)

(d) \( y = \int_6^x \frac{t^3}{8 + t^4} \, dt \)

(12) 4. Consider the function \( f(x) = e^{3x} \).

(a) Find an equation of the line tangent to the graph of \( y = f(x) \) at the point \( P = (1, e^3) \).

(b) Use linear approximation to get an estimate for the value of \( f(1.1) \).

(c) Use the second derivative to determine whether your estimate for (b) is likely to be high or low.
5. Consider the equation \( x^3 - 7x + 1 = 0 \).

   (a) Does this equation have a solution in the interval \([-3, -1]\)? Justify your answer.

   (b) Compute, using Newton’s method, the second approximation of a solution by starting with the first approximation \( x = -3 \). You do not need to simplify your final numerical answer.

6. Using the limit definition of the derivative, find the derivative of \( f(x) \) at \( x = 2 \) where

\[
f(x) = x^2 - 2x + 1.
\]

7. Find an equation of the line tangent to the graph of the function given implicitly by the equation \( 2x^2y - 3xy^2 = 14 \) at the point \((-2, 1)\).

8. An open-top box is to be made by cutting small congruent squares from the corners of a 6 cm \( \times \) 6 cm sheet of metal and bending up the sides. What is largest possible volume of such a box?

   ![Diagram of a box with squares cut out from the corners.]

9. Compute the following limits.

   (a) \( \lim_{x \to 2^+} \frac{x^2 - 4}{|x - 2|} \)

   (b) \( \lim_{x \to 1} \frac{x^2 - 4x + 5}{x^2 - 2} \)

   (c) \( \lim_{x \to \infty} \frac{\cos(x)}{1 + x^2} \)

   (d) \( \lim_{x \to 0} \frac{\cos(x) - 1}{e^x + e^{-x} - 2} \)
10. Find the global maximum and minimum values of the function

\[ f(x) = x^3 - 12x \]

on the interval \([-1, 3]\). You don’t have to sketch the graph but you must justify your answer.

Global maximum of \( f \): ____________  Global minimum of \( f \): ____________

11. Evaluate the following integrals.

(a) \[ \int_0^1 \frac{e^x}{\sqrt{1 + e^x}} \, dx \]

(b) \[ \int \left( \sqrt{x} + \frac{3}{\sqrt{x}} \right)^2 \, dx \]

(c) \[ \int_1^2 \frac{2x^2 - 1}{x} \, dx \]

12. A car travels on a restricted-access highway which is 100 miles long. The speed limit on the road is 60 miles per hour. Suppose that \( s(t) \), the car’s distance from the beginning of the road, is a differentiable function of the time \( t \). If the car enters the road at 9am and leaves the road at 10:15am at the other end of the highway, one and a quarter hours later, explain why the driver should expect to receive a speeding citation (ticket). Your explanation should be in complete English sentences. You should mention a specific result from this course and explain precisely why this result shows that the ticket is valid.

13. Let \( f(x) \) be a function defined over the interval \([0,4]\), whose graph is shown as below. The graph of \( f \) over \([1,3]\) is a semi-circle centered at \((2, 0)\) and of radius 1, the other pieces are straight lines.

![Graph of f(x)](image-url)
(a) Find $\int_0^4 f(x) \, dx$ geometrically.

(b) Compute the Riemann sum for $f(x)$ where $[0, 4]$ is subdivided into 4 subintervals of equal size and the sample points are the right endpoints.

(c) Let $F(x) = \int_0^x f(t) \, dt$. Find $F'(2)$ and $F'(3)$.

(10) 14. Determine the values of the parameters $a$ and $b$ such that the following function $f(x)$ becomes continuous and differentiable at $x = -1$:

$$f(x) = \begin{cases} 
  x^2 + 3x + b & \text{for } x > -1 \\
  ax & \text{for } x \leq -1
\end{cases}$$

(12) 15. Consider the finite region $A$ bounded by the “wedge” $y = |x|$ and the parabola $y = \frac{1}{4}x^2 + 1$.

(a) Draw a sketch of the region $A$. [Hint: The parabola fits “snuggly” into the wedge.]

(b) Express the area of $A$ in terms of one or more definite integrals.

(c) Compute the area of the region $A$ as a sum of fractions (you do not need to simplify the answer).

(10) 16. The minute hand of a large tower clock is 2 m long. At each full hour, the tip $T$ of the hand points to the center $C$ of the numeral XII. How fast is the distance between $T$ and $C$ changing when it is $9:20$ pm? [Hint: Sketch the triangle formed by $C$, $T$, and the center of the clock. Observe that it is isosceles.]
Show all your work. Full credit may not be given for an answer alone.
No books, no notes, no electronic devices.

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