Additional review problems for Math 152

The review materials for Exam 1 and the review materials for Exam 2 can be used to review for the Final Exam in Math 152. These materials did not include any problems from sections 11.3, 11.4, 9.1, 9.2, which are part of the course. For this reason, we are including the following additional review problems from 11.3, 11.4, 9.1, 9.2:

(1) Find the length of the cardioid \( r = 1 - \cos \theta \), \( 0 \leq \theta \leq 2\pi \).

(2) Find the area inside the cardioid \( r = 1 + \sin \theta \), \( 0 \leq \theta \leq 2\pi \).

(3) Find the center and radius of each of the two circles \( r = \sin \theta \), \( r = \cos \theta \).

(4) Prove that \( 2r = \sec(\theta + \pi/4) \) is a polar equation of a line in the \( xy \)-plane.

(5) Solve the following initial value problem: \( \frac{dx}{dt} = \tan x, \ x(0) = \pi/6 \).

(6) Solve the following initial value problem: \( (4 + x^3)^{1/2} \frac{dy}{dx} = (xy)^2, \ y(0) = -1 \).

(7) At time \( t = 0 \) minutes, a cup of coffee with a temperature of 100\(^\circ\) C is placed in a room with an ambient temperature of 25\(^\circ\) C. At time \( t = 10 \) minutes, the coffee temperature is 75\(^\circ\) C.
(a) What is the coffee temperature at time \( t = 20 \) minutes?
(b) At what time will the coffee reach a temperature of 50\(^\circ\) C?