Review Problems for Examination I

Mathematics 153, Fall 2002

1. Find the domain and range of the following functions:

(a) $f(x) = \sqrt{4-x}$ (b) $f(x) = \sqrt{4-x^2}$

2. Determine each of the limits:

(a) $\displaystyle\lim_{x\to 0} \frac{\sin(3x)}{\tan(5x)}$ (b) $\displaystyle\lim_{x\to\infty} (\sqrt{x^2+5x} - x)$ (c) $\displaystyle\lim_{x\to\infty} \frac{2x^2+3x+1}{\sqrt{9x^4+8x^3+3x+2}}$

3. Use the definition of the derivative directly to calculate $f'(x)$, where $f(x) = 1/\sqrt{x}$.

4. Sketch the graph of $y = f(x) = x^3 + \dfrac{1}{x}$.

(a) Determine those points x so that the tangent line to the curve at x is horizontal. Give exact values, not numerical approximations.

(b) Find the equation of the tangent line to the curve at the point $x = 1$ and draw the tangent line on the earlier graph.

5. Calculate the derivative of each fuction f:

(a) $f(x) = \sqrt{x^4+4x+4}$ (b) $f(x) = x^2\sin^3(x^4)$ (c) $f(x) = x^2 e^{-x^3}$

6. Find the inverse function g of the function $f(x) = e^x/(e^x+1)$.

7. Suppose that u is a positive real number and a, b, c are real numbers so that:

$\log_u(5) = a$, $\log_u(27) = b$, $\log_u(32) = c$. What is the numerical value of $u^{2a+(1/3)b-(2/5)c}$?

8. Suppose that f and g are functions. Asume that f is differentiable and $f(1) = 2, f'(1) = 4$ and that $g(x) = x^4 - x + 1$. Use this information to calculate:

(a) $(fg)'(1)$ (b) $(f/g)'(1)$ (c) $(f\circ g)'(1)$

9. With A and B constants, a function f is defined by: $f(x) = \begin{cases} 1/x + A, & \text{if } x < -1; \\ |x|, & \text{if } -1 \le x \le 1; \\ 1/x + B, & \text{if } 1 < x. \end{cases}$

(a) Find A and B so that f is continuous everywhere.
(b) Sketch the graph of $y = f(x)$.
(c) At which points does f fail to be differentiable? Explain the answer.

10. Let $f(x) = x^3 + x - 1$.

(a) Explain why f has a root in the interval $[0, 1]$.

(b) Let A be a constant and $g(x) = x^3 + x - 1 + Ax(x-1)(2x-1)$. Show that g has a root in the interval $[0, 1]$.

(c) By calculating $g(1/3)$ and $g(2/3)$, show that, if A is large enough, g has at least 3 roots in the interval $[0, 1]$.

11. Find a positive integer M so that if $x \ge M$, then $\dfrac{1}{x^2+x} \le \dfrac{1}{10^6}$.
Explain why the value of M has the required property.

12. A function $y = f(x)$ is defined implicitly by the equation: $x^3 + 3x^2y + y^3 = 5$.

(a) Find dy/dx in terms of x and y.

(b) Find the equation of the tangent line to the curve at the point $(1, 1)$.

(c) Locate the points on the curve where the tangent line is horizontal.

13. Find all points (a, b) on the parabola $y = x^2 - x$ so that the tangent line to the parabola at the point (a, b) contains the point $(2, 1)$.

14. Below are the graphs of two functions f and g. One of the functions is the derivative of the other. Determine which is the original function and which is its derivative, explaining your reasons.