

Supplementary Review Problems for Exam #1, Calculus 153

Fall 2002 (Alternate Version)

NOTE: Give **exact** answers unless approximations are specifically requested. Examples: π is also exact; 3.14 is a good approximation to it (so is $22/7$).

1. Find $\lim_{x \rightarrow 0} \frac{\sqrt{4x^2 + 9} - 3}{x^2}$ and $\lim_{x \rightarrow -2} \frac{x^2 + 6x + 8}{x + 2}$, if these limits exist.

2. Determine the domain and range of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Find a formula for the inverse function $f^{-1}(x)$, and determine its domain and range.

3. Suppose that f and g are differentiable functions such that $f(4) = -1$, $f'(4) = 2$, $g(4) = -3$, and $g'(4) = 5$.

Compute the derivatives of the following three functions at $x = 4$:

$$A(x) = 2f(x) - 3g(x); \quad B(x) = 3f(x)(g(x))^2; \quad C(x) = \left(\frac{f(x)}{g(x)}\right)^{1/3}$$

4. An object is moving along the y -axis, and its position at any time $t \geq 0$ is $y(t) = -t^2 + 3t - 2$. Using the definition of velocity as a limit, find the velocity of the object at the time $t = 3$.

Then graph the function $y = y(t)$ and find the equation of its tangent line at $(3, -2)$.

5. For each of the following three functions, find the *exact* coordinates of all the points on the graphs where the tangent line is horizontal.

$$f(x) = \frac{6x}{x^2 + 3}; \quad g(x) = x^2 e^{-3x}; \quad h(x) = \frac{\sin x}{2 + \cos x}$$

6. There is a line passing through the point $(2, 0)$ which is tangent to the graph of $y = x^{-2}$ somewhere in the first quadrant. Sketch the picture, find the slope of this line, and find the point of tangency.

7. For the graph of the function $f(x) = \frac{1 - e^{-x}}{2 - 3e^{-x}}$, give exact expressions for the location of all of its horizontal and vertical asymptotes, as well as all of its intercepts.

8. True or false? Give a brief explanation in each case.

a) The following function is continuous at $x = 1$: $f(x) = \begin{cases} 3x + 1, & \text{if } x \leq 1 \\ \frac{x^2 + x - 2}{x^4 - x^3}, & \text{if } x > 1 \end{cases}$

b) The equation $9x^2 - 4x^3 + 50 \cos x = -50$ is satisfied for some x between 0 and 5.

9. Let $f(x) = |\sin(x)|$, and set $g(x) = f'(x)$ for $x \in [-\pi, \pi]$, $x \neq 0$.

(a) Find a formula for $g(x)$ (valid on its domain)

(b) Graph $g(x)$

(c) Compute $\lim_{x \rightarrow 0^+} g(x)$, $\lim_{x \rightarrow 0^-} g(x)$, and $\lim_{x \rightarrow 0} g(x)$ (if these limits exist).

10. Consider the ellipse with equation: $x^2 - xy + y^2 = 3$.

(a) Find all horizontal and vertical tangent lines to this curve. There are four. Think of them as the sides of a rectangular box, and sketch this box, marking the points of tangency.

(b) Using your sketch as a guide, fill in the graph of the ellipse inside the box.

11. Below are the graphs of functions f and g . One of them is the derivative of the other. Decide which is which, state your decision, and justify your decision by comparing the two graphs at the points $x = -1$, $x = 0$, $x = 1$, $x = 2$ and $x = 3$ (and other points if you wish). Present your evidence clearly and concisely.

