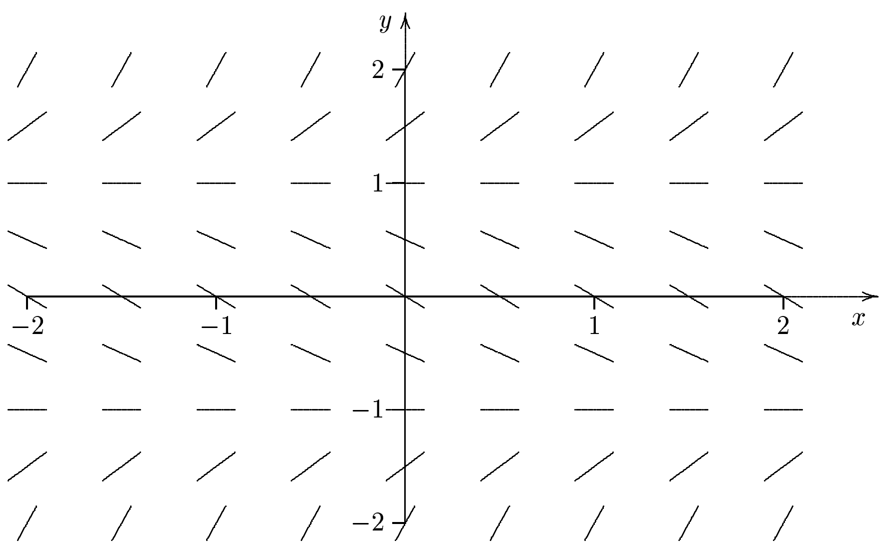
Thi is a set of selected review problems for the second exam. These problems hit some of the highlights of what will be covered, but do not cover everything that will be on the test. As usual, for the test you are responsible for knowing how to do the assigned problems and workshop problems. The test will cover material from the following sections of Stewart: 8.1,8.2,9.1,9.2,9.3,9.4, and Chapter 11, section 11.1–11.7.

1. a) Find the equation for the curve that passes through the point $x = 0$, $y = 2$ and whose slope at any point (x, y) is $y(y + 1)$.

2. Find all solutions of the differential equation $\dfrac{dy}{dx} = e^{-y}\cos x$.

3. Part of the direction field of the differential equation $\dfrac{dy}{dx} = y^2 - 1$ is shown on the axes below.

a) Suppose y_1, y_2 and y_2 are solutions to this differential equation with $y_1(0) = 1.5$, $y_2(0) = -.5$ and $y_3(0) = -1.5$. Make a rough sketch of the graphs of y_1, y_2, y_3 below (you do not need to find explicit formulas for these functions).



b) Suppose y is a solution to the differential equation that satisfies $-1 < y(0) < 1$. Use the direction field to find $\lim_{x\to+\infty} y(x)$.

4. Determine whether the given sequence is increasing, decreasing or not monotonic. Find the limit if the limit exists, or argue that the limit does not exist.

a) $a_n = \dfrac{n}{n^2 + n - 1}$ b) $a_n = \dfrac{n}{\ln(n + 1)}$ c) $a_n = 2 + (\dfrac{-2}{\pi})^n$

5. Let $a_1 = \sqrt{3}$, $a_2 = \sqrt{3\sqrt{3}}$, $a_3 = \sqrt{3\sqrt{3\sqrt{3}}}$, etc. Find a recursive equation for this sequence, i.e., show how to define a_{n+1} in terms of a_n for general n. Use this to show that if $a_n < 3$ then so is a_{n+1}, and conclude that the sequence is bounded. Show that $\ell = \lim_{n\to\infty} a_n$ exists and find ℓ.

6. Approximate the infinite series to an accuracy of 0.001. Explain your method and error bounds.

a) $\sum_{1}^{\infty} \frac{(-1)^n}{n^3+1}$

b) $\sum_{1}^{\infty} \frac{1}{n^3+1}$

7. Which series converge and which diverge? If a series converges, find its sum. If it diverges, explain why.

a) $\sum_{n=1}^{\infty} \frac{1}{(4n-3)(4n+1)}$

b) $\sum_{n=0}^{\infty} (\frac{e}{\pi})^n$

c) $\sum_{n=1}^{\infty} (-1)^{n+1} n$

8. Determine whether each of the following infinite series is absolutely convergent, convergent or divergent. Give full explanations of your answer, citing the test or technique used and the showing how it applies.

a) $\sum_{n=1}^{\infty} \frac{4n+7}{5n-1}$

b) $\sum_{2}^{\infty} \frac{1}{n \ln n}$

c) $\sum_{0}^{\infty} \frac{n^2}{5n^{7/2}-n+1}$

e) $\sum_{2}^{\infty} \frac{2^n}{3^n+n}$

f) $\sum_{2}^{\infty} \frac{(-1)^n}{n \ln n}$

g) $\sum_{2}^{\infty} \frac{2}{4n-1}$