

About the final.

Some review problems are posted on the course web site. These do not cover everything that will be tested, for instance calculating work, or slope fields. You should review all sections. There will not be a problem on calculating surface areas or average values. Additional good review problems and their solution may be found at <http://www.math.rutgers.edu/courses/152/152-Fall/152-f09/#Review>.

The pages that follow are the formula sheet you will get with the test. You will need to memorize the ratio test, how to find the radius of convergence of a power series, elementary facts about parametric equations. It would help to memorize the solution to the linear differential equation $y' = k(y - b)$, section 9.2, equation 2 (Newton's law of cooling, etc; you always derive this, but, on the test, you can just use the solution if it arises without re-deriving it. This is not an exhaustive list of the other facts you need to know. Practice working problems using the formula sheet, so that you can determine what else you want to memorize for the final. Review the previous exams, homeworks and workshops.

Formula Sheets

Error bounds for numerical estimates of integrals:

$$M_n = \Delta x [f(c_1) + \cdots + f(c_n)], \quad c_j = \frac{x_{j-1} + x_j}{2}$$

$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$$S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$\left| M_n - \int_a^b f(x) dx \right| \leq \frac{K_2(b-a)^3}{24n^2},$$

$$\left| T_n - \int_a^b f(x) dx \right| \leq \frac{K_2(b-a)^3}{12n^2},$$

$$\left| S_n - \int_a^b f(x) dx \right| \leq \frac{K_4(b-a)^5}{120n^4}.$$

Trigonometric identities:

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\sin(\theta + \psi) = \sin(\theta) \cos(\psi) + \sin(\psi) \cos(\theta)$$

$$\cos(\theta + \psi) = \cos(\theta) \cos(\psi) - \sin(\theta) \sin(\psi)$$

$$\sin(\theta) \cos(\psi) = \frac{1}{2} [\sin(\theta + \psi) + \sin(\theta - \psi)]$$

$$\cos(\theta) \cos(\psi) = \frac{1}{2} [\cos(\theta + \psi) + \cos(\theta - \psi)]$$

$$\sin(\theta) \sin(\psi) = \frac{1}{2} [\cos(\theta - \psi) - \cos(\theta + \psi)]$$

Integrals:

$$\int \sec u \, du = \ln \left| \sec u + \tan u \right| + c;$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} u + c;$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}(u/a) + c;$$

$$\int \sec u \tan u \, du = \sec u + c;$$

$$\int \tan u \, du = \ln \left| \sec u \right| + c;$$

$$\int \cot u \, du = \ln \left| \sin u \right| + c.$$

Taylor polynomial error bounds. Let $T_n(x)$ be the Taylor polynomial centered at a for f . Then

$$R_n(x) = f(x) - T_n(x) = \frac{1}{n!} \int_a^x (x-u)^n f^{(n+1)}(u) \, du.$$

Limit comparison for series. Suppose that $\{a_n\}$ and $\{b_n\}$ are positive sequences.

(a) If $\sum_1^\infty b_n$ converges and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} < \infty$, then $\sum_1^\infty a_n$ converges.

(b) If $\sum_1^\infty b_n$ diverges and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} > 0$, then $\sum_1^\infty a_n$ diverges.

Arc length: $s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt, \quad s = \int_{x_0}^{x_1} \sqrt{1 + (dy/dx)^2} dx.$

Area in polar coordinates: $A = \frac{1}{2} \int_{\theta_1}^{\theta_2} [f_2^2(\theta) - f_1^2(\theta)] d\theta.$

Maclaurin series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \text{for all } x.$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \text{for all } x.$$

$$\cos x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}, \quad \text{for all } x.$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{for } |x| < 1.$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \quad \text{for } x \text{ and } x = 1.$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad \text{for } |x| \leq 1.$$

$$(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n, \quad \text{for } |x| < 1, \text{ where } \binom{a}{0} = 1 \text{ and } \binom{a}{n} = \frac{a(a-1)\cdots(a-n+1)}{n!} \text{ for } n \geq 1.$$

Series error bounds. If $a_n = f(n)$ where f is positive and decreasing, then

$$0 < \sum_{n=1}^{\infty} a_n - \sum_{n=N+1}^{\infty} a_n \leq \int_N^{\infty} f(x) dx$$
$$\left| \sum_{n=1}^{\infty} (-1)^{n-1} a_n - \sum_{n=1}^N (-1)^{n-1} a_n \right| \leq a_{N+1}$$

Newton's Law of Cooling: $T' = -k(T - T_0).$

Hooke's Law: Restoring force on spring stretched x units past equilibrium is $F(x) = -kx.$