Formula Sheet for the first Midterm Exam

Error bounds: Let f be a given function on the interval $[a, b]$. Let K_2 be an upper bound on the values $\left|f''(x)\right|$ for $x \in [a, b]$, and let K_4 be an upper bound on $\left|f^{(iv)}(x)\right|$ for $x \in [a, b]$.

Let M_n, T_n, and S_n denote respectively the midpoint rule, trapezoid rule, and Simpson rule approximations of $\int_a^b f(x)\,dx$ using a partition of $[a, b]$ into n subintervals (S_n being defined only for even n). Then

$$\left|M_n - \int_a^b f(x)\,dx\right| \leq \frac{K_2(b-a)^3}{24n^2},$$

$$\left|T_n - \int_a^b f(x)\,dx\right| \leq \frac{K_2(b-a)^3}{12n^2},$$

$$\left|S_n - \int_a^b f(x)\,dx\right| \leq \frac{K_4(b-a)^5}{180n^4}.$$

Trigonometric identities:

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2} \qquad \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$$

$$\sin(\theta+\psi) = \sin(\theta)\cos(\psi)+\sin(\psi)\cos(\theta) \qquad \cos(\theta+\psi) = \cos(\theta)\cos(\psi)-\sin(\theta)\sin(\psi)$$

$$\sin(\theta)\cos(\psi) = \frac{1}{2}\left[\sin(\theta+\psi)+\sin(\theta-\psi)\right]$$

$$\cos(\theta)\cos(\psi) = \frac{1}{2}\left[\cos(\theta+\psi)+\cos(\theta-\psi)\right]$$

$$\sin(\theta)\sin(\psi) = \frac{1}{2}\left[\cos(\theta-\psi)-\cos(\theta+\psi)\right]$$

Integrals:

$$\int \sec u\,du = \ln\left|\sec u + \tan u\right| + c; \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} u + c;$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(u/a) + c; \qquad \int \sec u \tan u\,du = \sec u + c;$$

$$\int \tan u\,du = \ln\left|\sec u\right| + c; \qquad \int \cot u\,du = \ln\left|\sin u\right| + c.$$

$$\int \csc u\,du = \ln\left|\csc u - \cot u\right| + c;$$